Photonics, Spring 2025

Submit your answers as a PDF file via Google Classroom before deadline (17.03.2025 at 10.00). If problems, contact the course assistant joonas.mustonen@helsinki.fi.

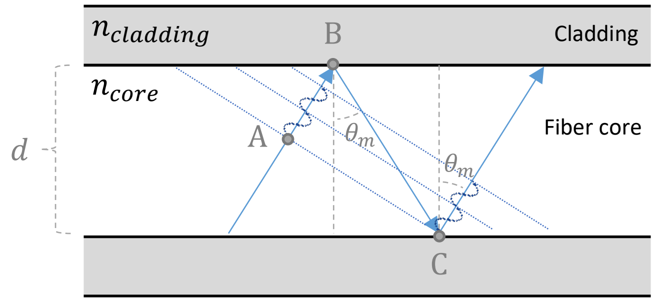
If you utilize LLM models as assistance in solving the task, please specify their usage at the end of your submission.

Exercise 6, 17.3.2025

1. **Waveguide condition** (2 points)

Two weeks ago (Exercise 4) we looked at optical fiber waveguides. Consider a single ray traveling along a waveguide undergoing total internal reflections at angle θ_m. Evidently for a coherent wave to propagate along the waveguide, the wavefronts from successive total internal reflections must interfere constructively. We have also learnt that TIR imposes a dependent phase-shift on the reflected ray, which is dependent on the incident angle θ_m.

Derive the waveguide condition for a slab waveguide (thickness d, refractive indices n_{core} and $n_{cladding}$, incident



angle with respect to the cladding is θ_m, vacuum wavelength λ_0, and the phase change upon reflection is ϕ_m):

$$\frac{2\pi n_{core} d}{\lambda_0} \cos\theta_m - \phi_m = m\pi$$

Hint: Consider the ray propagating along ABC. At point C there are two contributions to a phase shift relative to A; one arising from the optical path difference (OPD), and the other from reflections. How much phase shift would an OPD of λ cause? How about then an arbitrary OPD (ABC)? Note that: $\cos(2x) = 2\cos^2 x - 1$.

2. **Waveguide modes** (2 points)

Consider a planar dielectric waveguide with a core thickness of 20 µm, $n_{core} = 1.455$, $n_{cladding} = 1.440$, $\lambda = 900\ \mathrm{nm}$. Recall that the expression for phase-shift ϕ upon TIR, for a TE mode (s-polarized) is:

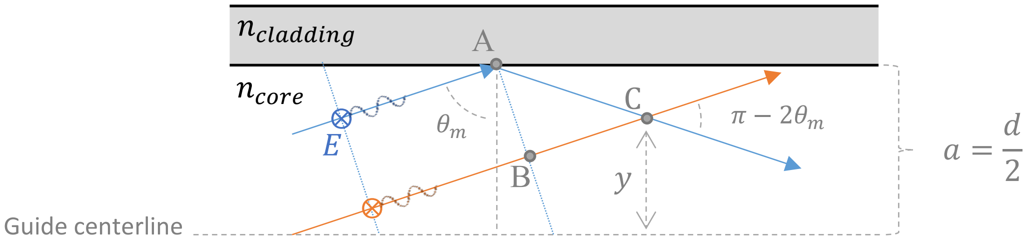
$$\tan\left(\frac{\phi_m}{2}\right) = \frac{\sqrt{\sin^2\theta_m - \left(\frac{n_{cladding}}{n_{core}}\right)^2}}{\cos\theta_m}$$

Numerically (graphically) compute the solution for θ_m and plot the results.

Hint: Using the waveguide condition (from Exercise 1), you can express ϕ_m as a function of θ_m and m. The left and right side are then both functions of θ_m. What do the intercepts of these functions correspond to? Note that the left function will be periodic, so solutions of odd and even values for m will overlap. The solutions for θ_i will be between 80° and 90°.

3. **Waveguide mode shapes** (4 points)

Let's continue the analysis from Exercise 1 to solve the actual electric field wavefunctions corresponding to different propagating modes. Consider now two identical parallel rays traveling in a waveguide.



a) Show that the phase difference between rays 1 and 2 at C is $\Delta\psi_m = \Delta\psi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$.
Hint: Use geometrical reasoning to derive ϕ_m as a function of θ_m, then use the waveguide condition to get rid of the explicit θ_m dependence. You'll probably need the identity: $\cos(2x) = 2\cos^2 x - 1$

b) Derive the field variation at C as a function of y (and m, a, ϕ_m) i.e., the wavefunction of the interfering wave in the form $E(y) = E_0 \cos(\omega t + \alpha)$. Note that the solved amplitude E_0 will be a cosine function arising from the geometry, however it has no time-dependence; $E_0 = E_0(\psi(y))$. Evidently also the phase shift $\alpha = \alpha(\psi(y))$. Note also there will be an arbitrary amplitude scaling factor.

Hint: Express the interfering waves as: $E(y) = A\cos(\omega t) + A\cos(\omega t + \Delta\psi_m(y))$. You'll probably need the following well-known trigonometric identity: $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$.

c) Plot the wave amplitude $E_0(y)$ for the first three TE modes, given that $a = 10$ µm, $\lambda = 1.3$ µm, $n_{core} = 1.455$ and $n_{cladding} = 1.440$.

Hint: You'll first have to express the phase difference $\Delta\psi$ as in part a). Then find θ_m using the phase difference formula and the waveguide condition, as in problem 2. Solve the phase differences ϕ_m by substituting the solved θ_m values into either equation. The solutions for θ_m will be between 80° and 90°.

d) Optical detectors measure intensity profiles. As we saw in Exercise 1, intensity is proportional to the electric field amplitude $I \propto E_0(y)^2$. Plot the intensity profile of the solved electric field.