Photonics, Spring 2025

Submit your answers as a PDF file via Google Classroom before deadline (20.01.2025 at 10.00). If problems, contact the course assistant joonas.mustonen@helsinki.fi.

If you utilize LLM models as assistance in solving the task, please specify their usage at the end of your submission.

Exercise 1, 13.01.2025

1. Maxwell's equations and wave equations (2 points)

Wave equations (describing the propagation of a certain quantity in time and space) are derived from the constitutive equations. In the case of electromagnetic waves, quantities **B** and **E** are coupled together via the following Maxwell's equations. Depending on the assumptions and boundary conditions, the derived wave equations may differ.

$$\nabla \cdot \mathbf{D} = \rho \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t})$$

$$\boldsymbol{D} = \epsilon \boldsymbol{E} + \boldsymbol{P} \qquad \boldsymbol{B} = \mu(\boldsymbol{M} + \boldsymbol{H})$$

a) Derive the linear **free space wave equation** for electric **E** and magnetic **B** fields from the Maxwell's equations.

Hint. Assumptions for the linear wave equation:
J= ρ =M=P=0

This implies there is no charge, and all material parameters correspond to the vacuum conditions, including no polarization.

b) Show that the following plane wave expression **E** is a solution to a wave equation, if the following velocity relation v is true:

$$\boldsymbol{E} = E_0 e^{i(kx-\omega t)}\hat{\boldsymbol{x}} \qquad v = \frac{c}{n}$$

Hint.

$$v = \frac{\omega}{k}$$

2. Non-linear optics (3 points)

a) Derive the non-linear wave equation for EM waves.
Hint. Change the assumption that the polarization vector **P** is zero and separate it as linear and non-linear parts
$P = P^L + P^{NL} \neq 0$

b) Describe situation, in which the aforementioned linear approximation is no longer valid, and non-linear approximation is required to model the phenomenon.
c) Name two applications of non-linear optics and explain them briefly.

3. Reflectance and transmittance (3 points)

Starting from the Fresnel equations:

$$reflected\ parallel: r_{||} = (\frac{E_r}{E_i})_{||} = \frac{n_2 \cos(\theta_i) - n_1 \cos(\theta_t)}{n_2 \cos(\theta_i) + n_1 \cos(\theta_t)}$$

$$transmitted\ parallel: t_{||} = (\frac{E_t}{E_i})_{||} = \frac{2n_1 \cos(\theta_i)}{n_2 \cos(\theta_i) + n_1 \cos(\theta_t)}$$

$$reflected\ perpendicular: r_{\perp} = (\frac{E_r}{E_i})_{\perp} = \frac{n_1 \cos(\theta_i) - n_2 \cos(\theta_t)}{n_1 \cos(\theta_i) + n_2 \cos(\theta_t)}$$

$$transmitted\ perpendicular: t_{\perp} = (\frac{E_t}{E_i})_{\perp} = \frac{2n_1 \cos(\theta_i)}{n_1 \cos(\theta_i) + n_2 \cos(\theta_t)}$$

a) Derive the expressions for the amplitude coefficients for normal incidence for both transverse magnetic (TM) and electric (TE) waves.

b) Derive the expression for coefficients of transmittance and reflectance at normal incidence:

$$R = \frac{I_r}{I_i} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

$$T = 1 - R = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

c) What is the percentage of reflected irradiance at an air-glass interface?

4. Gaussian beam (2 points)

a) What is Gaussian beam? Explain what divergence θ and Rayleigh range z_R of a Gaussian beam are.

b) Estimate θ and z_R of Gaussian beam from a HeNe laser with beam width $2w_0 = 1$ mm at z = 0. After propagating 10 m through vacuum, what will the beam width be?