Optics, Spring 2024

Submit your answers as a PDF file via Google Classroom before deadline (04.04.2024 at 10.00).
If problems, contact the course assistant joonas.mustonen@helsinki.fi.

Exercise 9 (Max points 10)

1. State of polarization (Ch 8.1) (4p.)

As discussed in previous exercises, polarization describes the orientation of the electric field's oscillations in electromagnetic waves. Therefore, by determining relative phases and magnitudes of orthogonal electric field components, state of polarization (SOP) can be computed.

a) Describe completely the state of polarization (in Hecht's convention, check the chapter) in following cases:

$$\boldsymbol{E} = \hat{x}E_0 \sin\left(2\pi\left(\frac{z}{\lambda} - ft\right)\right) - \hat{y}E_0 \sin\left(2\pi\left(\frac{z}{\lambda} - ft\right)\right)$$

$$\boldsymbol{E} = \hat{x}E_0 \cos(\omega t - kz) - \hat{y}E_0 \cos\left(\omega t - kz + \frac{\pi}{2}\right)$$

$$\boldsymbol{E} = \hat{x}E_0 \sin(\omega t - kz) - \hat{y}E_0 \sin\left(\omega t - kz + \frac{\pi}{4}\right)$$

b) What are Stokes parameters (in this context)?

2. Wave plates and polarizers (Ch 8.1, 9.7) (3p.)

a) (2p.) Plane wave $\mathbf{E} = E_0 \cos(\omega t - kx)$, $E_0 = [0, E_{0y}, E_{0z}]$ is incident at an ideal waveplate (phase change $\Delta\varphi$). The angle between the direction of polarization and the optical axis of the waveplate is α. The optical axis is parallel to the z axis. An ideal polarizer is placed after the wave plate, with the angle between the axis of polarization and z axis being β. Show that the intensity after the polarizer is:

$$I = I_0((\sin(\alpha)\sin(\beta) + \cos(\alpha)\cos(\beta))^2 + 4\sin(\alpha)\sin(\beta)\cos(\alpha)\cos(\beta)\sin^2(\frac{\Delta\phi}{2}))$$

Hint. Calculate time-averages yields to I.

b) Show that in the case of a half wave plate and $\alpha = \pi/4$, the output intensity is unchanged and the state of the polarization is preserved.

3. Birefringence (Ch 8.4, 8.7) (3p.)

a) Explain qualitatively what birefringence is.

b) A calcite crystal ($n_o = 1.6584$, $n_e = 1.4864$, $\lambda = 584.3$ nm) is polished in a way that the optical axis is perpendicular to the surface. A ray of light is incident at the surface at $\theta = 50°$. Compute the frequency, wavelength and angle refraction for o and e waves.

c) What is the minimum thickness for a quartz ($n_o = 1.5443$, $n_e = 1.5534$, $\lambda = 584.3$ nm) retarder if it is to be a quarter wave plate?