Optics, Spring 2024

Submit your answers as a PDF file via Google Classroom before deadline (08.02.2024 at 10.00).
If problems, contact the course assistant joonas.mustonen@helsinki.fi.

Exercise 2

1. Orthogonality of E, B and k vectors (3p.)

In case of electromagnetic waves, electric field **E**, magnetic field **B** and wavevector **k** are usually considered to obey orthogonality (which may or may not be true). Let's consider 3D EM plane waves as follows:

$$\boldsymbol{B}(\boldsymbol{r},t) = B_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

$$\boldsymbol{E}(\boldsymbol{r},t) = E_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

a) Derive the following expressions:

$$\nabla \cdot \mathbf{C} = \mathrm{i}\mathbf{k} \cdot \mathbf{C}$$

$$\nabla \times \mathbf{C} = i\mathbf{k} \times \mathbf{C}$$

in which **C** is **E** or **B**.

b) Recover the Maxwell's equations (exercise set 1) for plane waves by substituting the divergences and curls with expressions from the task a).

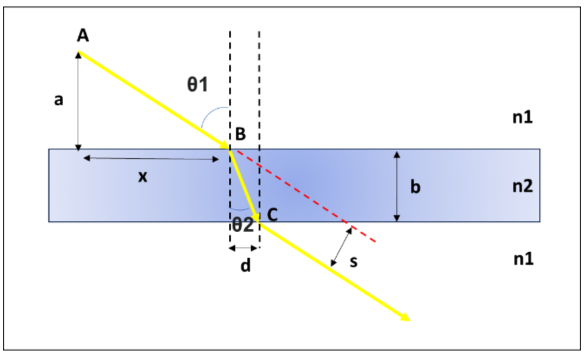
c) Show that **E**, **B** and **k** are a right-handed orthogonal triad.

2. Snell's law (3p.)

In ray optics, Fermat's principle applies, stating that light propagates the path which takes the least time. Therefore, the stationary point is found by the following spatial derivative of the propagation time t:

$$\frac{dt}{dx} = 0$$

Now, consider the following case, in which a planar glass plate (with refractive index n_2) is immersed in air (refractive index n_1). A light ray propagates to the glass plate from point A, the ray refracts at point B (interface between two media), and again refracts at point C, when a light comes out of the glass plate.



a) Derive Snell's law using Fermat's principle.

$$\theta_2 = arc\sin\left(\frac{n_1}{n_2}\sin(\theta_1)\right)$$

Hints. Here, x is the variable and a, b and d are constants.

$$v = \frac{x}{t};$$
$$v = \frac{c}{n}$$

b) Show that the ray from point A to B is parallel to the ray that exists the glass plate at point C.

c) Derive the expression for spacing s as a function of plate thickness b.

3. Reflection and transmission (3p.)

EM waves can be split into perpendicular components:

$E_\perp$ perpendicular to plane of incident, and

$E_{||}$ parallel to plane of incident.

Fresnel equations (derived in Hecht ch. 4.6.2 in 4th edition) describe reflected and transmitted electric field amplitudes for both perpendicular and parallel waves compared to incident amplitude E_i as follows.

$$reflected\ parallel: r_{||} = \left(\frac{E_r}{E_i}\right)_{||} = \frac{n_2\cos(\theta_i) - n_1\cos(\theta_t)}{n_2\cos(\theta_i) + n_1\cos(\theta_t)}$$

$$transmitted\ parallel: t_{||} = \left(\frac{E_t}{E_i}\right)_{||} = \frac{2n_1\cos(\theta_i)}{n_2\cos(\theta_i) + n_1\cos(\theta_t)}$$

$$reflected\ perpendicular: r_\perp = \left(\frac{E_r}{E_i}\right)_\perp = \frac{n_1\cos(\theta_i) - n_2\cos(\theta_t)}{n_1\cos(\theta_i) + n_2\cos(\theta_t)}$$

$$transmitted\ perpendicular: t_\perp = \left(\frac{E_t}{E_i}\right)_\perp = \frac{2n_1\cos(\theta_i)}{n_1\cos(\theta_i) + n_2\cos(\theta_t)}$$

a) Consider the simplest possible case, in which the incident ray is perpendicular to interface of two media. Calculate the aforementioned amplitude coefficients, if

$$\theta_i = \theta_r = \theta_t = 0°$$

b) Derive the expression for coefficients of transmittance and reflectance at normal incidence:

$$R = \frac{I_r}{I_i} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

$$T = 1 - R = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

In which I is irradiance, defined as follows:

$$I = \frac{c\epsilon}{2}E^2$$

c) What is the percentage of reflected irradiance at an air-glass interface (normal incidence, $n_{air} = 1$, $n_{glass} = 1.5$)?