Optics, Spring 2024

Submit your answers as a PDF file via Google Classroom before deadline (01.02.2024 at 10.00).
If problems, contact the course assistant joonas.mustonen@helsinki.fi.

Exercise 1

1. Complex number representation (2p.)

a) Show that multiplying a complex number z = x + yi by ± i is perpendicular to z.
b) Two waves ψ_1 and ψ_2 with the same amplitude A, frequency $\omega/2\pi$ and speed ω/k are overlapping in some region of space:

$$\psi_1(x,t) = Acos(kx + \omega t)$$

$$\psi_2(x,t) = Acos(kx - \omega t + \pi)$$

Show that the following equation is true and calculate the global maxima of the ψ:

$$\psi(x,t) = \sum_{i=1}^{2} \psi_i(x,t) = -2Asin(kx)sin(\omega t)$$

2. Electromagnetic quantities (1p.)

Define the following quantities, and their units.
a) Electric field **E**
b) Magnetic flux density **B**
c) Electric charge density ρ
d) Current density **J**
e) Permittivity ε
f) Permeability μ
g) Dielectric polarization density **P**
h) Electric displacement field **D**
i) Magnetic field strength **H**
j) Magnetization vector field **M**
k) Refractive index n

3. Linear electromagnetic wave equation (2p.)

Wave equations (describing the propagation of a certain quantity in time and space) are derived from the constitutive equations. In the case of electromagnetic waves, aforementioned quantities **B** and **E** are coupled together via the following Maxwell's equations. Depending on the assumptions and boundary conditions, the derived wave equations may differ.

$$\nabla \cdot \mathbf{D} = \rho \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathrm{t}} \qquad \nabla \times \mathbf{B} = \mu(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial \mathrm{t}})$$

$$\boldsymbol{D} = \epsilon \boldsymbol{E} + \boldsymbol{P} \qquad \boldsymbol{B} = \mu(\boldsymbol{M} + \boldsymbol{H})$$

a) Derive the linear free space wave equation for electric **E** and magnetic **B** fields from the Maxwell's equations.

Hint. Assumptions for the linear wave equation:
J= ρ =M=P=0

This implies there is no charge, and all material parameters correspond to the vacuum conditions, including no polarization.

b) Show that the following plane wave expression **E** is a solution to a wave equation, if the following velocity relation v is true:

$$\boldsymbol{E} = E_0 e^{i(kx-\omega t)}\hat{\boldsymbol{x}} \quad v = \frac{c}{n}$$

Hint.

$$v = \frac{\omega}{k}$$

4. Non-linear optics (3p.)

a) Derive the non-linear wave equation for EM waves.
Hint. Change the assumption that the polarization vector **P** is zero and separate it as linear and non-linear parts
$P = P^L + P^{NL} \neq 0$
b) Describe situation, in which the aforementioned linear approximation is no longer valid, and non-linear approximation is required to model the phenomenon.
c) Name two applications of non-linear optics and explain them briefly.