Photonics, Spring 2023

Exercises to be returned via Google Classroom by **14:00 next Tuesday 28.3.** For each problem, ½ a point will be awarded for an honest effort and 1 point for a well worked solution. Exercise sessions are held on Tuesdays from 16:00 - 18:00 at Chemicum A121.

Exercise 9, 28.3.2023

This week's exercises are a continuation of last week (**Optoelectronics and Photonics: Chapter 3**). A few fundamental concepts from last week relating to semiconductors are summarized here.

As a material is cooled, it's electrons evidently lose energy. 0 K then serves as a useful reference to determine an important energy level. The Fermi energy is defined as the difference between the lowest occupied and the highest empty energy states at 0 K. An electron will become conductive i.e., free, once its energy is raised above the Fermi energy. In semiconductors an energy gap (bandgap E_g) exists between free conduction electrons (conduction band CB) and bound electrons (valence band VB). Semiconductors with a relatively large bandgap $(E_c - E_f \gg k_B T)$ are referred to as nondegenerate; the number of electrons in the CB is far less than those in the VB. In contrast, degenerate semiconductors are so heavily doped that they behave more like a metal than a semiconductor.

Two important statistical concepts describe free electrons within an energy band. The first is density of states (DOS) $g(E)$, which basically represents the number of possible (allowed) electron energy states at a particular energy level. The Fermi-Dirac function $f(E)$ is then the probability of finding an electron in a quantum state corresponding to the energy E. Evidently at $E(f)$, $f(E_F) = 1/2$. The product of the two functions $g(E)f(E)$ then essentially represents the energy distribution of electrons i.e., how many electrons are found per unit energy per unit volume at a given energy E. The integral of this product then gives the electron concentration n in the energy range dictated by the integration limits. For nondegenerate semiconductors the Fermi-Dirac statistics can be reduced to Boltzmann statistics. For instance, the corresponding integral for electrons in the CB of a nondegenerate semiconductor gives the conduction electron concentration:

$$n = N_c \exp\left(-\frac{E_c - E_F}{k_B T}\right)$$

1. Extrinsic n-Si (2 points)

Extrinsic semiconductors are semiconductors that have been doped such that the concentrations of carriers of one polarity greatly outweigh that of the opposite polarity. Consider a Si crystal that has been doped n-type with 10^{17}cm^{-3} phosphorous donors. The electron drift mobility μ_e depends on the total concentration of ionized dopants N_d, as described in Table 1.
Hint: Check "*mass action law*" and Table 3.1. in the course textbook. When $N_d \gg n_i$, the conduction electron concentration will be nearly equal to $N_d \rightarrow n \approx N_d$. You can assume that doped Si is nondegenerate.

a) What is the conductivity σ of the crystal?

Using the mass action law, the hole concentration p can be expressed as a function of the material dependent intrinsic concentration n_i and the conduction electron concentration n:

$$np = N_C N_V \exp\left(-\frac{E_g}{k_B T}\right) = n_i^2$$

$$p = \frac{n_i^2}{n}$$

The conduction electron concentration is now equal to the n-type doping concentration $n = N_d$. The conductivity σ can then be expressed as the sum of contributions from each, weighted by their respective drift mobilities μ:

$$\sigma = en\mu_e + ep\mu_h = eN_d\mu_e + e\frac{n_i^2}{N_d}\mu_h \approx eN_d\mu_e$$

Using values from the table:

$$\sigma \approx 11.7\frac{\text{C}}{\text{cmVs}} \approx 11.7\frac{1}{\Omega\text{cm}}$$

b) Where is the Fermi level with respect to the intrinsic crystal ($E_{Fn} - E_{Fi}$)?
Hint: Find expressions for the intrinsic/dopant concentrations separately as functions of their respective Fermi levels, then take their ratio.

From table 3.1: $n_i = 10^{10}\text{cm}^{-3}$.
For intrinsic Si (undoped) using the mass action law (nondegenerate):

$$n_i = N_C e^{-\frac{E_C - E_{Fi}}{k_B T}}$$

And the for the doped Si:

$$N_d = n = N_C e^{-\frac{E_C - E_{Fn}}{k_B T}}$$

$$\frac{N_d}{n_i} = \frac{N_C e^{-\frac{E_C - E_{Fn}}{k_B T}}}{N_C e^{-\frac{E_C - E_{Fi}}{k_B T}}} = e^{\frac{E_{Fi} - E_{Fn}}{k_B T}}$$

$$\frac{E_{Fi} - E_{Fn}}{k_B T} = \ln\frac{N_d}{n_i} \rightarrow E_{Fn} - E_{Fi} = k_B T \ln\frac{N_d}{n_i} \approx 0.41 \text{ eV}$$

2. Compensation doping in n-type Si (5 points)

Compensation doping refers to the doping of a semiconductor with both donors and acceptors, which can lead to the reversal of the doping type. Consider an n-type Si sample that has been doped with 10^{16} phosphorous atoms per cm^{-3}.

a) What are the electron and hole concentrations?

From table 3.1: $n_i = 10^{10}\text{cm}^{-3}$

Electron concentration: $n = N_d = 10^{16}\text{ cm}^{-3}$

Hole concentration $p = \frac{n_i^2}{N_d} = 10\,000\text{ cm}^{-3}$

b) Calculate the room temperature conductivity of the sample.

$$\sigma = eN_d\mu_e + e\frac{n_i^2}{N_d}\mu_h \approx 1.92\frac{1}{\Omega\text{cm}}$$

c) Where is the Fermi level with respect to E_{Fi}?

$$E_{Fn} - E_{Fi} = k_B T \ln\frac{N_d}{n_i} \approx 0.35 \text{ eV}$$

d) If we now dope the crystal with an additional 10^{17} boron acceptors per cm^{-3}, what will be the conduction electron and hole concentrations?

$$N_a = 10^{17}\text{cm}^{-3} > N_d$$

Hole concentration: $p = N_a - N_d = 9 * 10^{16}\text{ cm}^{-3}$

Electron concentration: $n = \frac{n_i^2}{p} \approx 1110\text{ cm}^{-3}$

e) Where is the Fermi level corresponding to (d), with respect to E_{Fi}?

$$E_{Fp} - E_{Fi} = k_B T \ln \frac{p}{n_i} \approx -0.81 \text{ eV}$$

Dopant concentration (cm^{-3})	0	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}
GaAs, μ_e ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)	8500	-	8000	7000	5000	2400
GaAs, μ_h ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)	400	-	380	310	250	160
Si, μ_e ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)	1450	1420	1370	1200	730	280
Si, μ_h ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)	490	485	478	444	328	157

Table 1: Drift mobilities of conductivity electrons and holes (μ_e, μ_h) at various dopant concentrations.