Photonics, Spring 2023

Exercises to be returned via Google Classroom by **14:00 next Tuesday 11.4.** For each problem, ½ a point will be awarded for an honest effort and 1 point for a well worked solution. Exercise sessions are held on Tuesdays from 16:00 - 18:00 at Chemicum A121.

Exercise 11, 11.4.2023

1. Electrons in GaAs (3 points)

As a conduction band electron travels freely within the crystal lattice, it encounters a periodic potential energy that effectively manifests an inertial resistance to acceleration. The effective mass m_e^* is a quantum mechanical quantity that takes into account this resistance. The average kinetic energy of an electron is then simply $K = \frac{1}{2}m^*\langle v\rangle^2$. The root mean square velocity $\sqrt{\langle v^2\rangle}$ is called the thermal velocity v_{th}.

Recall that in Exercise 8 Problem 2c we derived, from the electron concentration distribution $n(E)$, that the average CB electron energy is $\frac{3}{2}k_BT$.

Consider then a nondegenerately doped GaAs semiconductor at room temperature (300 K). Let the electron effective mass m_e^* be 0.067 m_e, and the electron drift mobility $\mu_e = 8500\ \mathrm{cm^2V^{-1}s^{-1}}$.

a) Calculate the thermal velocity of the electrons in the CB.

Electrons travel in the crystal with an average energy of $\frac{3}{2}k_BT$:

$$\frac{1}{2}m_e^*v_{th}^2 = \frac{3}{2}k_BT$$

$$v_{th} = \sqrt{\frac{3k_BT}{m_e^*}} \approx 4.51 * 10^5\,\frac{\mathrm{m}}{\mathrm{s}}$$

Electrons travel FAST.

b) Given τ_e is the mean free time between electron scattering events (between electrons and lattice vibrations) and if $\mu_e = e\tau_e/m_e^*$, calculate τ_e.

$$\tau_e = \frac{\mu_e m_e^*}{e} \approx 3.24 * 10^{-13}\mathrm{s}$$

Electrons travel for extremely short intervals before scattering.

c) Calculate the drift velocity $v_d = \mu_e E$ of the CB electrons in an applied field E of $10^5\mathrm{Vm^{-1}}$. What is your conclusion?

$$v_d = \mu_e E = 8500\ \mathrm{cm^2V^{-1}s^{-1}} * 10^5\mathrm{Vm^{-1}} \approx 8.5 * 10^4\,\frac{\mathrm{m}}{\mathrm{s}}$$

The drift velocity is one order of magnitude smaller than the thermal velocity.

2. LED output spectrum (2 points)

a) The typical width of LED output spectrum in units of energy is $\approx 3k_BT$. Show that the linewidth $\Delta\lambda$ is approximately:

$$\Delta\lambda \approx \lambda^2\,\frac{3k_BT}{hc}$$

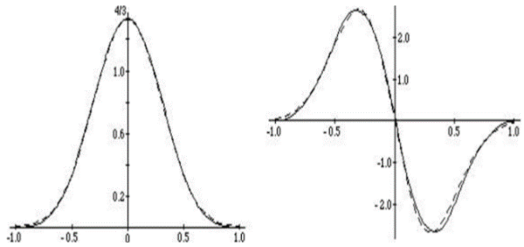
Hint: What is the energy of a photon in terms of its wavelength? What does the spectral distribution of a single color LED look like? What would the derivative of the spectral distribution look like near the half maximum? Could you then make an educated approximation of $d\lambda/dE$?

Photon energy:

$$E = hv = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E}$$

$$\frac{d\lambda}{dE} = -\frac{hc}{E^2} = -\frac{hc}{\left(\frac{hc}{\lambda}\right)^2} = -\frac{\lambda^2}{hc}$$

Looking at images of spectra for different colour LEDs, the spectral distribution is not far from a normal distribution. The derivative of a normal distribution in the range of the FWHM is then not far from linear:



$$\left|\frac{d\lambda}{dE}\right| \approx \frac{\Delta\lambda}{\Delta E} \rightarrow \Delta\lambda \approx \frac{\lambda^2}{hc}\Delta E = \frac{\lambda^2}{hc} 3k_B T$$

b) Keeping in mind that conduction band electrons have an energy distribution, the lowest conduction band energy level and the highest valence band energy level form the familiar bandgap energy E_g. The maximum electron concentration in the CB is at $\frac{1}{2}k_B T$ above E_g.

In LEDs, the higher energy photons corresponding to this maximum can become reabsorbed in the material producing high energy electrons and holes. These photogenerated high energy electrons and holes can recombine, emitting secondary photons with lower energies. For this reason, the photon emission spectrum of LEDs is wider ($\approx 3k_B T$) than what would be expected from the electron density distribution.

The bandgap of a GaAs LED is 1.42 eV at 300 K. The bandgap energy decreases with temperature as $dE_g/dT \approx -4.5 \times 10^{-4}\text{eVK}^{-1}$. Compute the change in the peak (central) wavelength of emitted photons if the temperature changes 10°C.

$$E_g(300\text{ K}) = 1.42\text{ eV}$$

Evidently there is a linear relationship between BG energy and T:

$$\frac{dE_g}{dT} \approx constant = \frac{\Delta E_g}{\Delta T} = -4.5 \times 10^{-4}\text{eVK}^{-1}$$

$$\Delta E_g = -4.5 \times 10^{-4}\text{eVK}^{-1}\Delta T$$

$$\Delta E_g(10°\text{C}) = -4.5 \times 10^{-4}\text{eVK}^{-1}10\text{ K} = -4.5 \times 10^{-3}\text{eV}$$

$$E_g(310\text{ K}) = E_g(300\text{ K}) + \Delta E_g = 1.42\text{eV} - 4.5 \times 10^{-3}\text{eV} = 1.416\text{ eV}$$

The center wavelength is given by:

$$\lambda_0 = \frac{ch}{E} = \frac{ch}{E_g + \frac{1}{2}k_B T}$$

$$\Delta\lambda = \left(\frac{ch}{E_g(T = 310\text{ K}) + \frac{1}{2}k_B 310\text{ K}}\right) - \left(\frac{ch}{E_g(T = 300\text{ K}) + \frac{1}{2}k_B 300\text{ K}}\right)$$

$$\Delta\lambda = ch\left(\frac{1}{1.416\text{ eV} + \frac{1}{2}k_B 310\text{ K}} - \frac{1}{1.42\text{ eV} + \frac{1}{2}k_B 300\text{ K}}\right)$$

$$\Delta\lambda \approx 2.2\text{ nm}$$

The shift in center wavelength is rather small over a 10 °C temperature variation.

3. LED linewidth (2 points)

Suppose that the width of the output spectrum is given by $\Delta E_{photon} = mk_B T$, where m is a numerical constant.

a) Compute $\Delta\lambda$ as a function of m.

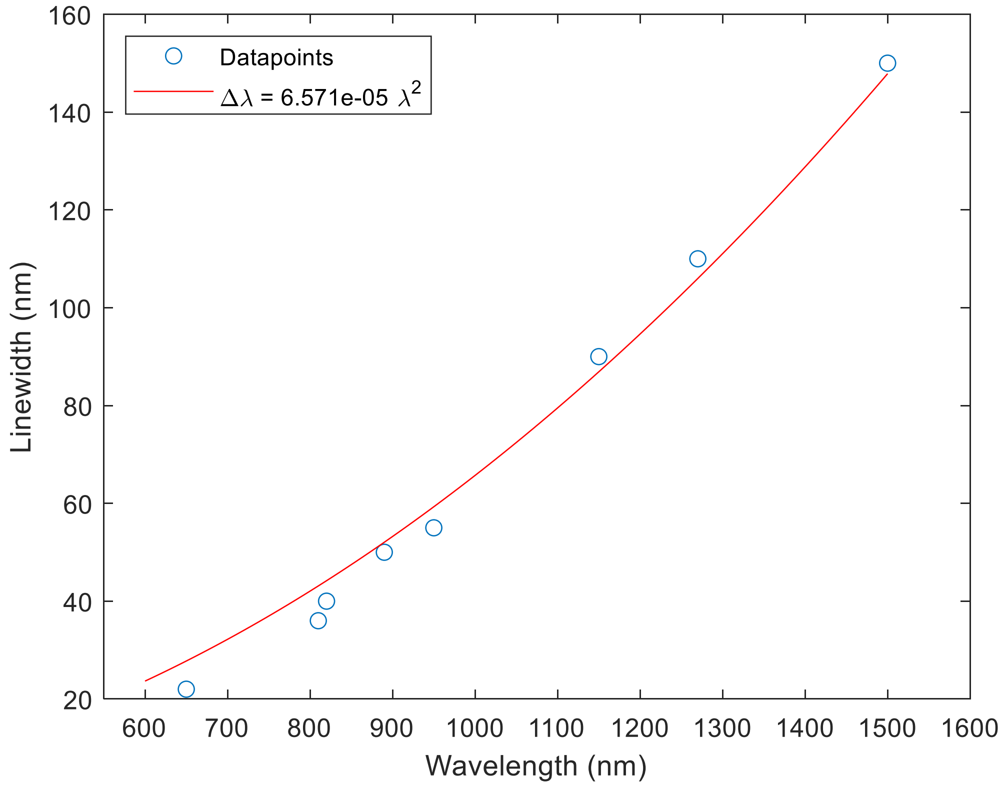
Using the result from problem 2:

$$\Delta\lambda \approx \frac{\lambda^2}{hc}\Delta E = \lambda^2 \frac{k_B T}{hc} m$$

b) The table below shows peak (central) wavelengths and linewidths for various LED junction materials. Plot the data in the table and compute m. Assume room temperature.

λ [nm]	650	810	820	890	950	1150	1270	1500
$\Delta\lambda_{1/2}$ [nm]	22	36	40	50	55	90	110	150
Material	AlGaAs	AlGaAs	AlGaAs	GaAs	GaAs	InGaAsP	InGaAsP	InGaAsP

Fitting a parabolic function $y = a\lambda^2$ to the data set:



$$a = (6.571 * 10^{-5} \pm 0.322 * 10^{-5})\ \text{nm}^{-1} = \frac{mk_BT}{hc}$$

$$m = \frac{ahc}{k_BT} = \frac{6.571 * 10^{-5}\text{nm}^{-1} * 6.626 * 10^{-34}\text{m}^2\text{kg} * \text{s}^{-1} * 3 * 10^8\text{m} * \text{s}^{-1}}{1.381 * 10^{-23}\text{m}^2\text{kg} * \text{s}^{-2}\text{K}^{-1} * 300\text{K}}$$

$$m \approx 3.153 \pm 0.155$$

4. LED-Fiber coupling efficiency (1 points)

200 μW of optical power is coupled into a multimode step index fiber from a SLED when $I = 75$ mA and $V = 1.5$ V. Compute the overall efficiency of power conversion from input electrical power to optical output power.

$$\eta_{PCE} = \frac{P_{optical}}{IV} = \frac{200\ \mu\text{W}}{75\ \text{mA} * 1.5\ \text{V}} \approx 0.00178 \approx 0.18\%$$