

# Photonics, Spring 2023

Exercises to be returned via Google Classroom by **14:00 next Tuesday 28.3**. For each problem,  $\frac{1}{2}$  a point will be awarded for an honest effort and 1 point for a well worked solution. Exercise sessions are held on Tuesdays from 16:00 - 18:00 at Chemicum A121.

## Exercise 9, 28.3.2023

This week's exercises are a continuation of last week ([Optoelectronics and Photonics: Chapter 3](#)). A few fundamental concepts from last week relating to semiconductors are summarized here.

As a material is cooled, its electrons evidently lose energy. 0 K then serves as a useful reference to determine an important energy level. The Fermi energy is defined as the difference between the lowest occupied and the highest empty energy states at 0 K. An electron will become conductive i.e., free, once its energy is raised above the Fermi energy. In semiconductors an energy gap (bandgap  $E_g$ ) exists between free conduction electrons (conduction band CB) and bound electrons (valence band VB). Semiconductors with a relatively large bandgap ( $E_c - E_f \gg k_B T$ ) are referred to as nondegenerate; the number of electrons in the CB is far less than those in the VB. In contrast, degenerate semiconductors are so heavily doped that they behave more like a metal than a semiconductor.

Two important statistical concepts describe free electrons within an energy band. The first is density of states (DOS)  $g(E)$ , which basically represents the number of possible (allowed) electron energy states at a particular energy level. The Fermi-Dirac function  $f(E)$  is then the probability of finding an electron in a quantum state corresponding to the energy  $E$ . Evidently at  $E(f)$ ,  $f(E_f) = 1/2$ . The product of the two functions  $g(E)f(E)$  then essentially represents the energy distribution of electrons i.e., how many electrons are found per unit energy per unit volume at a given energy  $E$ . The integral of this product then gives the electron concentration  $n$  in the energy range dictated by the integration limits. For nondegenerate semiconductors the Fermi-Dirac statistics can be reduced to Boltzmann statistics. For instance, the corresponding integral for electrons in the CB of a nondegenerate semiconductor gives the conduction electron concentration:

$$n = N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right)$$

### 1. Extrinsic n-Si (2 points)

Extrinsic semiconductors are semiconductors that have been doped such that the concentrations of carriers of one polarity greatly outweigh that of the opposite polarity. Consider a Si crystal that has been doped n-type with  $10^{17} \text{cm}^{-3}$  phosphorous donors. The electron drift mobility  $\mu_e$  depends on the total concentration of ionized dopants  $N_d$ , as described in Table 1.

**Hint:** Check “*mass action law*” and Table 3.1. in the course textbook. When  $N_d \gg n_i$ , the conduction electron concentration will be nearly equal to  $N_d \rightarrow n \approx N_d$ . You can assume that doped Si is nondegenerate.

- What is the conductivity  $\sigma$  of the crystal?
- Where is the Fermi level with respect to the intrinsic crystal ( $E_{Fn} - E_{Fi}$ )?

**Hint:** Find expressions for the intrinsic/dopant concentrations separately as functions of their respective Fermi levels, then take their ratio.

## 2. Compensation doping in n-type Si (5 points)

Compensation doping refers to the doping of a semiconductor with both donors and acceptors, which can lead to the reversal of the doping type. Consider an n-type Si sample that has been doped with  $10^{16}$  phosphorous atoms per  $\text{cm}^{-3}$ .

- What are the electron and hole concentrations?
- Calculate the room temperature conductivity of the sample.
- Where is the Fermi level with respect to  $E_{Fi}$ ?
- If we now dope the crystal with an additional  $10^{17}$  boron acceptors per  $\text{cm}^{-3}$ , what will be the conduction electron and hole concentrations?
- Where is the Fermi level corresponding to (d), with respect to  $E_{Fi}$ ?

Dopant concentration ( $\text{cm}^{-3}$ )	0	$10^{14}$	$10^{15}$	$10^{16}$	$10^{17}$	$10^{18}$
GaAs, $\mu_e$ ( $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ )	8500	-	8000	7000	5000	2400
GaAs, $\mu_h$ ( $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ )	400	-	380	310	250	160
Si, $\mu_e$ ( $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ )	1450	1420	1370	1200	730	280
Si, $\mu_h$ ( $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ )	490	485	478	444	328	157

Table 1: Drift mobilities of conductivity electrons and holes ( $\mu_e, \mu_h$ ) at various dopant concentrations.