Photonics, Spring 2023

Exercises to be returned via Google Classroom by **14:00 next Tuesday 21.3.** For each problem, ½ a point will be awarded for an honest effort and 1 point for a well worked solution. Exercise sessions are held on Tuesdays from 16:00 - 18:00 at Chemicum A121.

Exercise 8, 21.3.2023

This week's exercises have to do with some fundamental concepts relating to semiconductors, including electron energy levels and the accompanying statistics (**Optoelectronics and Photonics: Chapter 3**).

1. Fundamental concepts relating to semiconductor (4 points)

In the context of semiconductors, verbally define/describe the following:

a) Density of states (DOS) $g(E)$

Density of states (DOS) $g(E)$, which basically represents the number of possible (allowed) electron energy states at a particular energy level.

b) Fermi-Dirac function $f(E)$

The Fermi-Dirac function $f(E)$ is then the probability of finding an electron in a quantum state corresponding to the energy E. Evidently at $E(f)$, $f(E_F) = 1/2$.

c) The product $g(E)f(E)$ and it's integral

The product of the two functions $g(E)f(E)$ then essentially represents the energy distribution of electrons i.e., how many electrons are found per unit energy per unit volume at a given energy E. The integral of this product then gives the electron concentration n in the energy range dictated by the integration limits.

d) Degenerate and nondegenerate semiconductors, and how Boltzmann statistics relates to the latter

Semiconductors with a relatively large bandgap $(E_c - E_f \gg k_B T)$ are referred to as nondegenerate; the number of electrons in the CB is far less than those in the VB. In contrast, degenerate semiconductors are so heavily doped that they behave more like a metal than a semiconductor. For nondegenerate semiconductors the Fermi-Dirac statistics can be reduced to Boltzmann statistics. For instance, the corresponding integral for electrons in the CB of a nondegenerate semiconductor gives the conduction electron concentration:

$$n = N_c \exp\left(-\frac{E_c - E_F}{k_B T}\right)$$

2. Electrons in the CB of a nondegenerate semiconductor (X points)

a) Consider the energy distribution of electrons $n_E(E)$ in the conduction band (CB). Assuming that the density of state $g_{CB}(E) \propto \sqrt{E - E_c}$ and using Boltzmann statistics $f(E) \approx \exp[-(E - E_F)/k_B T]$ show that the energy distribution of the electrons in the CB can be written as $n_x(x) = C\sqrt{x}\exp(-x)$ where $x = (E - E_c)/k_B T$, is the electron energy in terms of $k_B T$ measured from E_C, and C is a constant at a given temperature (independent of E).

Hint: Introduce an arbitrary constant based on the proportionality argument for $g_{CB}(E)$. Try then to arrive at the desired form by introducing multiplier terms in the form a/a.

Since we know the proportionality of the density of states, we can express it with an arbitrary constant:

$$g_{CB}(E) \propto \sqrt{E - E_C} \rightarrow g_{CB}(E) = A\sqrt{E - E_C}$$

The energy distribution of electrons is then the product of the density of states and the probability distribution of electrons in these states:

$$n_E(E) = g_{CB}(E) f(E) = A\sqrt{E - E_C}\, e^{-\frac{E - E_F}{k_B T}}$$

$$= A\sqrt{E - E_C}\, e^{-\frac{E}{k_B T}} e^{\frac{E_F}{k_B T}}$$

We want to arrive at an exponent with $E - E_c$, so let's introduce that:

$$n_E(E) = A\sqrt{E - E_C}\, e^{-\frac{E}{k_B T}} e^{\frac{E_F}{k_B T}} e^{-\frac{E_C}{k_B T}} e^{\frac{E_C}{k_B T}}$$

$$= A\sqrt{E - E_C}\, e^{-\frac{E_C - E_F}{k_B T}} e^{-\frac{E - E_C}{k_B T}}$$

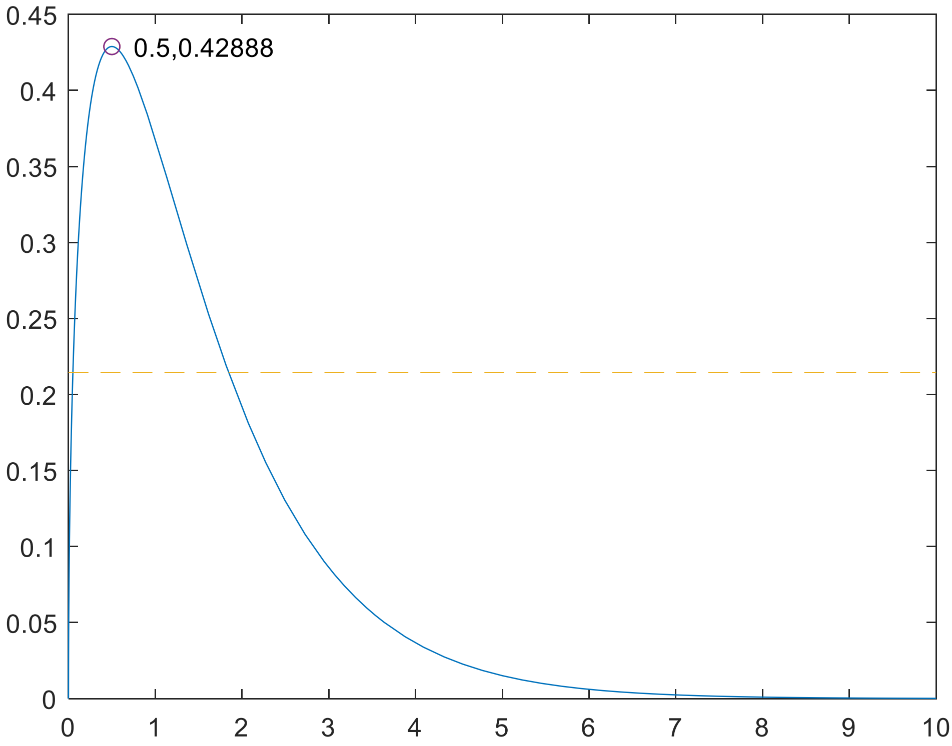
This is almost the sought after form, lacking only the $k_B T$ factor:

$$n_E(E) = A\sqrt{k_B T}\sqrt{\frac{E - E_C}{k_B T}}\, e^{-\frac{E_C - E_F}{k_B T}} e^{-\frac{E - E_C}{k_B T}}$$

$$\rightarrow n_x(x) = C\sqrt{x}e^{-x}$$

where $x = \frac{E - E_C}{k_B T}$, $C = A\sqrt{k_B T}\, e^{-\frac{E_C - E_F}{k_B T}}$.

b) Setting arbitrarily $C = 1$, plot $n(x)$. Where is the maximum and what is the FWHM of the curve?



The maximum is at $x = 0.5$ and the FWHM is ≈ 1.79

c) Using numerical integration, show that the average electron energy in the CB is $\frac{3}{2}k_BT$ (above the bottom of the conduction band).

Hint: The approach is similar to what we have seen with time-averaged quantities. Now instead of time, the normalizing quantity is the electron concentration $n_E(E)$. Recall that the total energy in a given state is $En_E(E)$. What should the integral expression for the average then look like? Try to then express it as a function of x (as in part (a)).

The average energy can be solved by integrating the energy in a given state multiplied by the corresponding electron concentration, and divided by the total number of electrons:

$$E_{avg} = \frac{\int_{E_C}^{E_C+\chi} En_E(E)dE}{\int_{E_C}^{E_C+\chi} n_E(E)dE}$$

Based on part (a), we can express energy concentration as a function of $x = \frac{E-E_C}{k_BT}$:

$$n_E(E) \to n(x) = C\sqrt{x}e^{-x}$$

And the energy corresponding to a given state is then:

$$x = \frac{E - E_C}{k_BT} \to E(x) = xk_BT + E_C$$

The total energy in a given state is then:

$$En_E(E) = (xk_BT + E_C)C\sqrt{x}e^{-x}$$

The integral can then be written as:

$$E_{avg} = \frac{\int_0^\infty (xk_BT + E_C)C\sqrt{x}e^{-x}dx}{\int_0^\infty C\sqrt{x}e^{-x}dx}$$

$$= \frac{\int_0^\infty (xk_BT + E_C)\sqrt{x}e^{-x}dx}{\int_0^\infty \sqrt{x}e^{-x}dx}$$

Integrating this with MATLAB:

$$= \frac{3}{2}k_BT + E_C$$

d) Show that the maximum in the energy distribution is at $x = 1/2$ or at $E_{max} = \frac{1}{2}k_BT$.

Easy enough:

$$x = \frac{1}{2} \to \frac{1}{2} = \frac{E - E_C}{k_BT} \to E = E_C + \frac{1}{2}k_BT$$

The k_BT term corresponds to the maximum energy (above the bottom of the conduction band).