Photonics, Spring 2023

Exercises to be returned via Google Classroom by **14:00 next Tuesday 14.3.** For each problem, ½ a point will be awarded for an honest effort and 1 point for a well worked solution. Exercise sessions are held on Tuesdays from 16:00 - 18:00 at Chemicum A121 – **<u>no sessions on 7.3.</u>**

Exercise 7, 14.3.2023

A rather lengthy discussion is provided here concerning dispersion. There is an important distinction to be made between dispersion due to material properties, and dispersion due to a wave-guide structure.

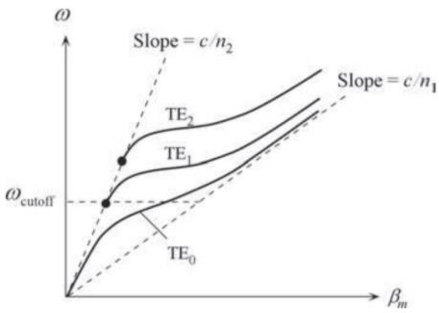
As discussed in previous weeks, light sources always have a finite bandwidth $\Delta\lambda$ i.e., they emit a range of frequencies. As a result, a light pulse will spread in space as it travels. Evidently this spread can also be considered as a temporal one $\Delta\tau$. Dispersion is often expressed as a spread per unit length:

$$\frac{\Delta\tau}{L} = |D_m|\Delta\lambda$$

where L is the distance traveled, D_m is the material dispersion coefficient. The dispersion coefficient is approximately given by the second derivative of the refractive index, and is evaluated at the center wavelength:

$$D_m \approx -\frac{\lambda}{c}\left(\frac{d^2n}{d\lambda^2}\right)$$

Wave-guide structures introduce dispersion that is distinct from material dispersion. Consider a wave reflecting back and forth from the walls of a planar waveguide, as we have done in previous weeks. The wave-vector of a certain propagating mode m can be split into components parallel and perpendicular to the wave-guide, of which evidently the forward propagating mode is of interest: $\beta_m = k\sin\theta_m$. The allowed incident (to the waveguide wall) angle θ_m is determined by the waveguide condition, which depends on both wavelength (frequency) and the waveguide properties (n_1, n_2, d). Therefore, the wave-vector is also a function of both *frequency* and *the properties of the wave-guide*. Given the wave-guide properties, ω and β_m can be calculated for each mode, producing a nonlinear relationship between ω and β_m, which can be plotted as a dispersion diagram (ω vs. β_m):



Recall then that group velocity $v_g = d\omega/dk$ indicates the velocity at which energy (information) travels, which for any given mode is the gradient of the dispersion curve. The group velocity therefore depends on both the frequency and the waveguide properties. This is an important result, which implies that even if the refractive indices n_1, n_2 are wavelength-independent, the waveguide will introduce dispersion. This dispersion, similarly to material dispersion, is characterized by the waveguide dispersion coefficient:

$$\frac{\Delta\tau}{L} = |D_w|\Delta\lambda$$

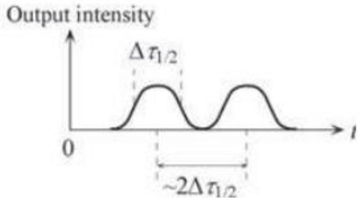
1. **Dispersion and bitrate** (2 point)

a)

i) The dispersion (material + waveguide) of a short light pulse propagating in a fiber is characterized by the temporal broadened pulse width $\Delta\tau = \Delta\tau_{FWHM} = \Delta\tau_{1/2}$. Estimate the maximum return to zero bitrate (0 should be read between two successive 1's) corresponding to $\Delta\tau_{1/2}$.

Hint: While any sensible estimate will do, the sought for answer is simple; consider two dispersed pulses side by side – at what separation in terms of $\Delta\tau_{1/2}$ is there a "sufficient" minimum between the two?

A sensible estimate for distinguishable 1 bits (with a 0 return) would be $2\Delta\tau_{1/2}$



The bitrate would then be the inverse of this time separation:

$$B \approx \frac{1}{2\Delta\tau_{1/2}}$$

ii) In Exercise 4, Problem 3 we calculated the temporal broadening of an optical pulse in a 1 km long pure silica fiber due to material dispersion: $\Delta\tau_{1/2} = 1.68\ \text{ns}$. As a reminder, the solution was arrived at by deriving the group index N_g, approximating $\frac{dt}{d\lambda} \approx \frac{\Delta t}{\Delta\lambda}$ and noting that the propagation time $t = L/v_g$. Calculate then in this case the corresponding maximum bitrate limited by material dispersion.

$$B \approx \frac{1}{2\Delta\tau_{1/2}} \approx 298\ \text{Mbits/s}$$

b) If the received light pulse intensity has a Gaussian shape, with a root-mean-square dispersion (temporal standard deviation) σ, show that the maximum bitrate is:

$$B \approx \frac{0.25}{\sigma}$$

Hint: As in part a), make a justified estimate of how far the pulses should be spaced. The factor 0.25 should give you a pretty solid hint.

For a Gaussian distribution, 95% of the intensity is contained within 2σ. Then a separation of 4σ would mean only approximately 5% of the optical power is present at the overlapped pulses. Seems like a sensible condition for bitrate:

$$\Delta\tau = 4\sigma \rightarrow B = \frac{1}{4\sigma} = \frac{0.25}{\sigma}$$

2. **Multimode fiber** (5 point)

A few weeks ago (Exercise 4) we discussed some characteristic quantities describing waveguides, namely numerical aperture NA, and V-number. To recap, the numerical aperture NA is a dimensionless number characterizing the range of acceptance angles α of an optical system:

$$NA = n_0 \sin\alpha_{max} = \sqrt{n_1^2 - n_2^2}$$

The V-number, or normalized frequency, is related to the (number of) propagating modes m. For a given wavelength the V-number depends on the waveguide geometry. For a step-index fiber, the V-number is straightforward to derive from the waveguide condition and applying the TIR condition ($\sin\theta_m > \sin\theta_c$):

$$m \le \frac{2V_{number} - \phi_m}{\pi} \quad , \quad V_{number} = \frac{\pi D}{\lambda_0} NA$$

For single-mode fibers $V_{number} \le 2.405$, above which the number of modes rises sharply. A good approximation for the number of modes in a step-index multimode fiber is given by:

$$M \approx \frac{V_{number}^2}{2}$$

Consider a multimode fiber operating at $\lambda = 850$ nm, with a core diameter of 100 µm, core refractive index of 1.4750, and a cladding refractive index of 1.4550. Calculate:

a) The V-number for the fiber and estimate the number of modes

$$V_{number} = \frac{\pi D}{\lambda_0}\sqrt{n_1^2 - n_2^2}$$

$$= \frac{\pi 100\ \mu\text{m}}{850\ \text{nm}}\sqrt{1.4750^2 - 1.4550^2} \approx 89.47$$

Approximate number of modes:

$$M \approx \frac{V^2}{2} = \frac{89.47^2}{2} \approx 4002$$

b) The wavelength beyond which the mode becomes single-mode (cut-off wavelength)

$$V_{number} \le 2.405 \ge \frac{\pi D}{\lambda_0}\sqrt{n_1^2 - n_2^2}$$

$$\lambda_0 \ge \frac{\pi D}{2.405}\sqrt{n_1^2 - n_2^2}$$

$$\lambda_0 \ge \frac{\pi\ 100\ \mu\text{m}}{2.405}\sqrt{1.4750^2 - 1.4550^2} \approx 31.6\ \mu\text{m}$$

c) Numerical aperture

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.4750^2 - 1.4550^2} \approx 0.24$$

d) Maximum acceptance angle

$$NA = n_0 \sin\alpha_{max} = \sqrt{n_1^2 - n_2^2}$$

$$\alpha_{max} \approx \sin^{-1}\sqrt{1.4750^2 - 1.4550^2} \approx 14°$$

e) Modal dispersion per unit length $\Delta\tau/L$ and hence the bitrate-distance product ($B \cdot L$). Assume a gaussian pulse.

Hint: Consider the temporal separation of the slowest and fastest modes, then refer to the graph on the previous page. The (return-to-zero) bitrate requires a separation of 4 standard deviations: $B = 0.25/\sigma$. The relationship between the modal dispersion $\Delta\tau_{1/2}$, which is defined at the full-width-half-maximum intensity, and the standard deviation of a gaussian pulse is: $\sigma \approx 0.425\ \Delta\tau_{1/2}$.

The time traveled of a single mode is $\tau = \frac{L}{c}$. The difference in the fastest and slowest modes will give us the temporal broadening (modal dispersion):

$$\Delta\tau = \frac{L}{v_{gmin}} - \frac{L}{v_{gmax}}$$

$$\frac{\Delta\tau}{L} = \frac{1}{v_{gmin}} - \frac{1}{v_{gmax}}$$

$$= \frac{n_1 - n_2}{c} = \frac{1.475 - 1.455}{c} \approx 6.67 * \frac{10^{-11}\text{s}}{\text{m}} = 66.7\frac{\text{ns}}{\text{km}}$$

The bitrate is then:

$$B = \frac{0.25}{\sigma} = \frac{0.25}{0.425\ \Delta\tau_{1/2}} = \frac{0.25}{0.425 * 66.7 \frac{\text{ns}}{\text{km}} * L}$$

The bitrate-distance product is then:

$$B * L = \frac{0.25}{0.425 * 66.7 \frac{\text{ns}}{\text{km}} * L} * L$$

$$\approx 8.82 \frac{\text{Mbit}}{\text{s}} * \text{km}$$

3. Single mode fiber (5 points)

Consider a fiber with a 86.5% SiO_2 – 13.5% GeO_2 core of diameter 8 µm and a refractive index of 1.468, and a cladding refractive index of 1.464. The core is operated with a laser source of $\lambda = 1300\ \text{nm}$ and a half-maximum width of 2 nm. Calculate:

a) The V-number for the fiber – is this a single mode fiber?

$$V = \frac{\pi D}{\lambda_0} \sqrt{n_1^2 - n_2^2}$$

$$= \frac{\pi\ 8\ \mu\text{m}}{1.3\ \mu\text{m}} \sqrt{1.468^2 - 1.464^2} \approx 2.094$$

The V-number is smaller than 2.405, so it is a single mode fiber.

b) The wavelength below which the fiber becomes multi-mode (cut-off wavelength)

$$V > 2.405 < \frac{\pi D}{\lambda_0} \sqrt{n_1^2 - n_2^2}$$

$$\lambda_0 < \frac{\pi\ 8\mu\text{m}}{2.405} \sqrt{1.468^2 - 1.464^2} \approx 1130\ \text{nm}$$

c) Numerical aperture

$$NA = \sqrt{1.468^2 - 1.464^2} \approx 0.11$$

d) Maximum acceptance angle

$$\alpha_{max} \approx \sin^{-1} \sqrt{1.468^2 - 1.464^2} \approx 6.2°$$

e) The total (material and waveguide) dispersion, and hence estimate the bitrate-distance product ($B \cdot L$). ($D_m = -7.5\ \text{ps km}^{-1}\ \text{nm}^{-1}, D_w = -5\ \text{ps km}^{-1}\ \text{nm}^{-1}$).
The total dispersion is the sum of material and waveguide contributions:

$$\frac{\Delta\tau}{L} = \frac{\Delta\tau_m}{L} + \frac{\Delta\tau_w}{L} = |D_m|\Delta\lambda + |D_w|\Delta\lambda = (|D_m| + |D_w|)\Delta\lambda$$

$$= (7.5\ \text{ps km}^{-1}\ \text{nm}^{-1} + 5\text{ps km}^{-1}\ \text{nm}^{-1})\ 2\ \text{nm} = 2.5 * \frac{10^{-14}\text{s}}{\text{m}} = 25\ \text{ps/km}$$

The bitrate is then:

$$B = \frac{0.25}{\sigma} = \frac{0.25}{0.425\ \Delta\tau_{1/2}} = \frac{0.25}{0.425 * 25\ \text{ps/km}} \approx 23.5 \frac{\text{Gbit}}{\text{s}} * \text{km}$$

4. **Graded index fiber** (3 points)

If a fiber has an axial index profile of:

$$n = n_1[1 - 2\Delta(r/a)^\gamma] \quad r > a$$

$$n = n_2 \quad r \geq a$$

where $\gamma = 2(1-\Delta)$, and $\Delta = (n_1 - n_2)/n_1$ is the normalized index difference, the dispersion per unit length is given by:

$$\frac{\sigma}{L} \approx \frac{n_1}{20\sqrt{3}c}\Delta^2$$

Consider a graded index fiber, with $n_1 = 1.480, n_2 = 1.460,\ \lambda = 1.3\ \mu\text{m}, a = 25\ \mu\text{m}$.

a) Compute the bitrate-length-product.

The dispersion per unit length is:

$$\frac{\sigma}{L} \approx \frac{n_1}{20\sqrt{3}c}\Delta^2 = \frac{n_1}{20\sqrt{3}c}\left(\frac{n_1 - n_2}{n_1}\right)^2$$

$$= \frac{1}{20\sqrt{3}cn_1}(n_1 - n_2)^2$$

$$= \frac{1}{20\sqrt{3}c1.48}(1.48 - 1.46)^2 \approx 2.6 * 10^{-14}\frac{\text{s}}{\text{m}} = 0.026\frac{\text{ns}}{\text{km}}$$

Bitrate-distance product:

$$BL \approx \frac{0.25L}{\sigma} = \frac{0.25}{0.026\frac{\text{ns}}{\text{km}}} \approx 9.62\frac{\text{Gbit}}{\text{s}}\text{km}$$

b) Compute the equivalent value of a multimode step index fiber.

Multimode:

$$\Delta\tau_{MM} = \frac{L}{v_{gmin}} - \frac{L}{v_{gmax}}$$

$$\frac{\Delta\tau_{MM}}{L} = \frac{1}{v_{gmin}} - \frac{1}{v_{gmax}} = \frac{n_1 - n_2}{c} = \frac{1.480 - 1.460}{c} = 6.67 * 10^{-11}\frac{\text{s}}{\text{m}}$$

$$BL_{MM} \approx \frac{0.25 * L}{0.425\frac{\Delta\tau_{\frac{1}{2}}}{L}} = \frac{0.25}{0.425 * 6.67 * 10^{-11}\frac{\text{s}}{\text{m}}} \approx 8.8\frac{\text{Mbit}}{\text{s}}\text{km}$$

c) Compare the results, also to the result of the single mode fiber from 3 e). Which one is best, why?

The bitrate of the graded index fiber is about 1000 greater than the multimode fiber, and about half of that of the singlemode fiber. The graded index minimizes intermodal dispersion (difference in travel time between modes) and is capable of propagating around half the modes that a step index multimode fiber does.

Strictly speaking the optimal choice depends on the application, but generally the graded index fiber can carry multiple simultaneous signals at an excellent bitrate. Also it's worth mentioning that the graded index fiber is designed for a single wavelength, and intermodal dispersion worsens rapidly at different wavelengths; profile index $\gamma = \gamma(n_1, n_2) \rightarrow n_1, n_2 = n(\lambda)$.