Photonics, Spring 2023

Exercises to be returned via Google Classroom by **14:00 next Tuesday 4.4.** For each problem, ½ a point will be awarded for an honest effort and 1 point for a well worked solution. Exercise sessions are held on Tuesdays from 16:00 - 18:00 at Chemicum A121.

Exercise 10, 4.4.2023

This week's exercises continue from the topics covered in the previous two weeks, moving on to the combination of two oppositely doped semiconductors i.e., the formation of pn-junctions. Let's recap a few relevant concepts, an introduce some new ones.

The fermi energy level is a reference energy level defined at 0 K, where the probability of finding an electron with a corresponding energy is 0.5; it is in the middle of the last occupied state and first unoccupied state. For materials without dopant atoms, the number of positive and negative charge carriers is equal. As a material's temperature is increased, more electrons are excited to the conduction band, leaving holes in the valence band. Therefore, the intrinsic charge carrier concentration n_i gives the concentration of both electrons and holes $(n_i = n = p)$ in non-doped materials at a given temperature.

For nondegenerate (relatively large bandgap) semiconductors the Fermi-Dirac statistics describing charge carrier probability distributions approximately reduce to the more simple Boltzmann statistics. The concentration of negative (n) and positive (p) charge carriers can then be expressed as:

$$n \approx N_C \exp\left(-\frac{E_C - E_F}{k_B T}\right) \qquad p \approx N_V \exp\left(-\frac{E_F - E_V}{k_B T}\right)$$

The coefficients N_C and N_V are temperature dependent material constants referred to as *effective density of states*, at the edge of the conduction and valence bands, respectively. This *effective density* is conceptually rather simple; take all energy states in the conduction (valence) band, replace them with an effective concentration N_C (N_V) at energy E_C (E_V) and multiply by the Boltzmann probability function $\exp(-(E_C - E_F)/k_B T)$, and you'd get the concentration of electrons at energy E_C (E_V).

The mass action law states that the rate of a given chemical reaction is proportional to the product of masses of the reacting substances. For semiconductors, this results in a useful form of the mass action law: at thermal equilibrium, the product of the number of electrons in the conduction band and holes in the valence band is constant regardless of the doping of the material:

$$np = N_C N_V \exp\left(-\frac{E_g}{k_B T}\right) = n_i^2$$

Particularly for doped nondegenerate semiconductors, often the dopant (donor or acceptor) concentration is far greater than the intrinsic concentration N_d or $N_a \gg n_i$. In such cases then the number of conduction/valence (majority) charge carriers is approximately equal to the number of dopants, which greatly simplifies calculations. Calculating the concentration of the opposite (minority) charge carriers is then straightforward using the mass action law.

When a p- and n-type semiconductor are bonded, they form a pn-junction (chapter 3.9). The p-side then has a lowered Fermi-energy level; extra holes (unoccupied energy levels) are introduced above the valence band. The n-side, respectively, has a raised Fermi-energy level; extra electrons are repelled from the valence band closer to the conduction band, introducing extra occupied energy levels. Assuming the joined material is in equilibrium without introducing energy into the system (light, heat, voltage, etc.) the Fermi levels of both sides must be equivalent. There is then a deformation of the valence and conduction bands between the two semiconductors, referred to as the **depletion region**, or **space charge layer** (**SCL**). This region forms instantaneously as the two materials are combined; electrons (holes) diffuse from the n-type (p-type) to the p-type (n-type) across the junction. These opposite charge carriers recombine, depleting the respective regions of their characteristic majority charge carriers. The result is a net positive charge in the depleted n-type and a negative charge in the depleted p-type, giving rise to an electric field. This electric field attempts to drift the charge carriers back into their respective sides. Eventually the diffusion and drift of charge carriers reaches equilibrium, manifesting a **built-in voltage** V_0 across the junction that prevents further net charge transfer. The depletion zone is characterized by it's characteristic width:

$$V_0 = -\frac{1}{2}E_0 W_0 = -\frac{eN_a N_d W_0^2}{2\epsilon(N_a + N_d)}$$

Since the total charge must be equal on both sides of the junction, a relation can be drawn between the concentration-width product of each side:

$$N_a W_{p0} = N_d W_{n0}$$

Following from Boltzmann statistics the built-in voltage is given by:

$$V_0 = \frac{k_B T}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

When an external voltage V is introduced across the junction such that the positive terminal is connected to the p-side and the negative to the n-side (forward bias), the potential barrier V_0 will be reduced by V. As a consequence charge diffusion across the junction is facilitated, promoting the injection of excess minority charge carriers; holes into the n-region and electrons into the p-region. A concentration gradient of charge carriers is introduced also outside the depletion zone, which results in the diffusion of minority charge carriers across the junction. As these minority charge carriers travel into the recipient semiconductor, they will eventually recombine with an opposite majority charge carrier. Evidently this diffusion phenomenon contributes to the current, which is characterized by the corresponding recombination time τ, or the respective diffusion length $L = \sqrt{D\tau}$. The diffusion current density is given by the **Shockley equation**:

$$J_D = J_{D,hole} + J_{D,elec} = en_i^2\left(\frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a}\right)\left[\exp\left(\frac{eV}{k_{BT}}\right) - 1\right]$$

In addition to diffusion of minority charge carriers in the neutral regions (outside the depletion zone), recombination also occurs within the depletion zone. Therefore, the external current source must also replenish charge carriers lost to recombination within the depletion zone. Each side of the depletion region can be characterized by their widths W_n, W_p and the mean electron/hole recombination times τ_e, τ_n. The resulting current density due to recombination within the depletion region is given by:

$$J_{recom} = \frac{en_i}{2}\left(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_n}\right)\exp\left(\frac{eV}{2k_B T}\right)$$

1. **Si pn-junction** (3 points)

An abrupt Si pn-junction has acceptor dopants ($N_a = 10^{18}\text{cm}^{-3}$) on one side and donor dopants ($N_d = 5 \times 10^{15}\text{cm}^{-3}$) on the other side.

a) Calculate how much the Fermi levels are shifted from the intrinsic level, at room temperature, in the p and n regions.
Hint: Find expressions for the intrinsic/dopant concentrations separately as functions of their respective Fermi levels, then take their ratio. A similar exercise was done last week.

b) Draw an equilibrium energy band diagram for the junction and determine the contact potential energy eV_0 from the diagram.
Hint: Refer to chapter 3.9 of the course textbook.

c) Compare the result with the potential calculated from $V_0 = \frac{k_B T}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$.

2. **Si pn-junction** (2 points)

Consider a long Si pn junction diode ($n_i \approx 10^{10}\text{cm}^{-3}$) with an acceptor doping $N_a = 10^{18}\text{cm}^{-3}$ on the p-side and donor concentration N_d on the n-side. The diode is forward biased with a voltage of 0.6 V across it. The diode cross-sectional area is 1 mm^2. The minority carrier recombination time τ depends on the dopant concentration N_{dopant} through the following approximate empirical relation:

$$\tau \approx \frac{5 \times 10^{-7}}{1 + 2 \times 10^{-17} N_{dopant}} \frac{\text{s}}{\text{cm}^3}$$

The Einstein relation defines the general form of the diffusion coefficient as a function of mobility μ and temperature T:

$$D = \mu k_B T \quad [\text{m}^2/\text{s}]$$

For charged particles, electrical mobility is defined as $\mu_q = \mu q$.

Dopant concentration (cm^{-3})	0	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}
GaAs, μ_e ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)	8500	-	8000	7000	5000	2400
GaAs, μ_h ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)	400	-	380	310	250	160
Si, μ_e ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)	1450	1420	1370	1200	730	280
Si, μ_h ($\text{cm}^2\text{V}^{-1}\text{s}^{-1}$)	490	485	478	444	328	157

Table 1: Drift mobilities of conductivity electrons and holes (μ_e, μ_h) at various dopant concentrations.

a) Suppose that $N_d = 10^{15}\text{cm}^{-3}$ and $N_a = 1 \times 10^{18}\text{cm}^{-3}$. The depletion layer now essentially extends into the n-side, and in order to determine the current the minority carrier recombination times must be taken into account. Calculate the diffusion and recombination contributions to the total diode current. What is your conclusion?
Hint: The necessary equations are introduced in the beginning of these exercises, although it's up to you to figure out how to get from current density to current. It's probably worth converting to SI units to help out with eliminating units during calculations.

b) Suppose that $N_d = N_a$. Then the depletion region width W extends equally to both sides and $\tau_e = \tau_h$. Calculate the diffusion and recombination contributions to the diode current given that $N_a = N_d = 10^{18}\text{cm}^{-3}$. What is your conclusion?

3. Si pn-junction bias (2 points)

An abrupt Si pn-junction ($A = 10^{-4}\text{cm}^2$) has the following properties at 300K:

p	n
$N_a = 10^{17}\text{cm}^{-3}$	$N_d = 10^{15}\text{cm}^{-3}$
$\tau_n = 0.1\ \mu\text{s}$	$\tau_p = 10\ \mu\text{s}$
$\mu_p = 200\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$	$\mu_n = 1300\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$
$\mu_n = 700$	$\mu_p = 4500$

Calculate the current over the junction when it is forward (0.5 V) or reverse (-0.5 V) biased.
Hint: Based on exercise 2, since $N_a \gg N_d$ you can assume that $J_{diff} \gg J_{recomb} \rightarrow J \approx J_{diff}$.