Photonics, Spring 2023

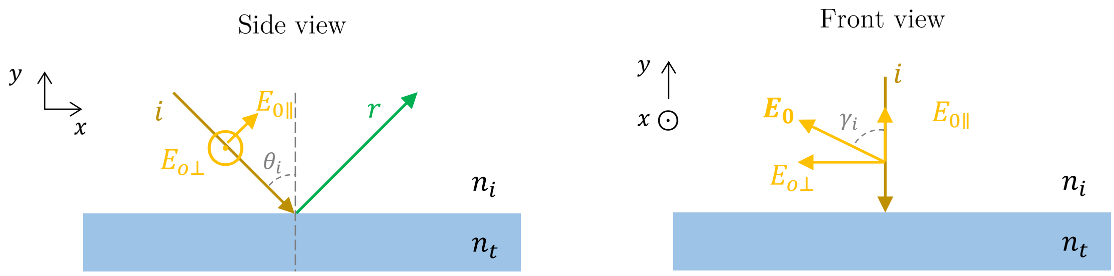
Exercises to be returned via Google Classroom by **14:00 next Tuesday 31.1.** (link sent by Joonas Mustonen via email). **Include your name and student number in the returned exercises**. For each problem, ½ a point will be awarded for an honest effort and 1 point for a well worked solution.

Exercise sessions are held on Tuesdays from 16:00 - 18:00 at Chemicum A121.

Exercise 2, 24.1.2023

1. Reflectance of polarized light (1 point)

A linearly polarized plane wave is incident on an interface between two linear, homogenous non-magnetic di-electric media. The angle between the plane of incidence (xy-plane) and direction of polarization is γ_i and the angle of incidence is θ_i. The reflectance components of the p- and s-polarized components are $R_{\parallel}$ and $R_{\perp}$, respectively. Write an expression for the total reflectance R.
Hint: start by writing $R = I_r/I_i$, where I_r is the total reflected intensity ($I = \frac{c\epsilon}{2}E_0^2$).



2. Reflectance of natural light (1 points)

Natural, or unpolarized light, is such that the angle γ_i of problem 1 changes rapidly and randomly. Derive an expression for the reflectance of natural light R_n in terms of $I_{\parallel}$ and $I_{\perp}$.
Hint: problem 1 should give the reflectance R as a function of γ_i. Since the polarization angle now varies randomly, take the time average of that result: $R_n = \langle R \rangle = \langle f(\gamma)R_{\parallel} + g(\gamma)R_{\perp} \rangle$.

3. Reflection and transmission at a semiconductor-semiconductor interface (4 points)

A light wave with a free space wavelength of 890 nm that is propagating in GaAs becomes incident on AlGaAs. The refractive indices of GaAs and AlGaAs are 3.60 and 3.30, respectively.

a) Consider normal incidence. What are the reflection and transmission coefficients and the reflectance and transmittance?

b) What is the Brewster angle (polarization angle θ_p) and the critical angle θ_c for total internal reflection for the wave incident on the GaAs/AlGaAs interface?

c) What is the reflection coefficient and the phase change in the reflected wave when the angle of incidence is $\theta_i = 79°$?

d) What is the penetration depth of the evanescent wave into the AlGaAs when $\theta_i = 79°$ and when $\theta_i = 89°$? Does the result make sense?

4. Evanescent wave (2 points)

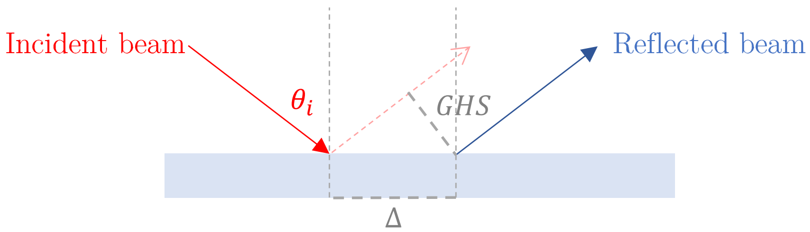
Total internal reflection (TIR) of a plane wave from a boundary between a dense (n_1) and a denser (n_2) medium is accompanied by an evanescent wave propagating in medium 2. Derive the corresponding plane wave wavefunction and discuss how its magnitude varies with penetration into medium 2. The general wavefunction is:

$$\boldsymbol{E} = \boldsymbol{E_0} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

Hint: draw a simple ray diagram of the general case of incident and transmitted rays (without TIR). Express the wave vector in terms of its orthogonal components (x, y). When $\theta_i > \theta_c$ the transmission angle becomes complex, where the imaginary part describes the Evanescent wave; the imaginary term allows for exponential decay of the resulting Evanescent wave. The goal is then to express both components of the wave vector as functions of θ_i, which remains real-valued. Using definitions of the refractive index $n = c/v = \lambda_0/\lambda$, the wavenumber $k = 2\pi/\lambda$ and Snell's law, strive to express the wave vector components as functions of the real-valued θ_i and readily known parameters n and λ_0.

5. Goos-Hänchen shift (2 points)

The Goos-Hänchen shift can be thought of as an addition to the law of reflection ($\theta_i = \theta_r$). Experiments conducted by Goos and Hänchen in the mid 1900's demonstrated that during total internal reflection, the reflection of a real beam with a finite width will appear to be displaced along the interface i.e., a spatial shift Δ occurs between the incident and reflected rays:



The Goos-Hänchen shift GHS is defined as the lateral shift with respect to reflected beam, and the displacement along the surface Δ is (http://www.scholarpedia.org/article/Goos-Hänchen_effect):

$$\Delta = -\frac{\lambda_0}{2\pi n_i \cos\theta_i}\frac{\partial\phi(\theta_i)}{\partial\theta_i}$$

ϕ is the phase shift. A ray of light travelling in a glass medium ($n_1 = 1.460$) becomes incident on another glass medium ($n_2 = 1.430$). The free-space wavelength of the light ray is $\lambda_0 = 850$ nm and the angle of incidence is $\theta_i = 85°$. Estimate the Goos-Hänchen shift (GHS) in the reflected wave for the parallel and perpendicular field components. What can you conclude about the reflected waves, and would there be some applications for this effect?