Photonics, Spring 2023

Exercises to be returned via Google Classroom by **14:00 next Tuesday 24.1.** (link sent by Joonas Mustonen via email). **Include your name and student number in the returned exercises**. For each problem, ½ a point will be awarded for an honest effort and 1 point for a well worked solution.

Exercise sessions are held on Tuesdays from 16:00 - 18:00 at Chemicum A121.

Exercise 1, 17.1.2023

1. Maxwell's equations and waves (1 point)

Derive the free space wave equation for electric and magnetic fields from Maxwell's equations.

2. Reflectance and transmittance (3 points)

Starting from the Fresnel equations:

$$r_{\parallel} = \left(\frac{E_{0r}}{E_{0i}}\right)_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} \qquad t_{\parallel} = \left(\frac{E_{0t}}{E_{0i}}\right)_{\parallel} = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$r_{\perp} = \left(\frac{E_{0r}}{E_{0i}}\right)_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad t_{\perp} = \left(\frac{E_{0t}}{E_{0i}}\right)_{\perp} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

a) Derive the expressions for the amplitude coefficients for normal incidence for both transverse magnetic (TM) and electric (TE) waves.

b) Derive the expression for coefficients of transmittance and reflectance at normal incidence:

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

$$T = \frac{4n_1 n_2}{(n_2 + n_1)^2}$$

c) What is the percentage of reflected irradiance at an air-glass interface (normal incidence, $n_{air} = 1, n_{glass} = 1.5$)?

3. Total internal reflection (3 points)

A ray of light is incident at an interface between two linear dielectric media ($n_1 = 1.45, n_2 = 1.33$, vacuum wavelength $\lambda = 1.064\ \mu m$)

a) Calculate the minimum angle for total internal reflection

b) Calculate the phase change in reflection, when $\theta_i = 70°$

c) Calculate the penetration depth, when $\theta_i = 70°$

4. Antireflection coatings (2 points)

a) Consider three dielectric media, with flat and parallel boundaries with refractive indices n_1, n_2 and n_3. Show that for normal incidence the reflection coefficient between layer 1 and 3 is the same as between layers 2 and 3 if $n_2 = \sqrt{n_1 n_3}$

b) What should be the refractive index and thickness of an antireflection coating designed to operate at $\lambda_0 = 1.064\ \mu m$ on a BK-7 lens in air?

5. Gaussian beam (1 point)

Estimate the divergence Θ and Rayleigh range z_R of a Gaussian beam from a HeNe laser with beam width $2w_0 = 1\ mm$ at $z = 0$. After traversing 10 m through vacuum, what will the beam width be?