530192 “Photonics in semiconductors”, (5 op / 3 ov), period III and IV – Spring 2017

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Course webpage:
http://electronics.physics.helsinki.fi/teaching/photonics-in-semiconductors-2017/

High-Power Semiconductor Lasers

nLIGHT

LOHJA, FINLAND *(LIEKKI)*

nLIGHT’s manufacturing facility in Lohja, Finland is approximately 1.900 m^2 with a clean room environment, including areas for preform manufacturing, fiber drawing, fiber measurement and testing, and optical engine assembly.

Visit to nLight

23.03.2017

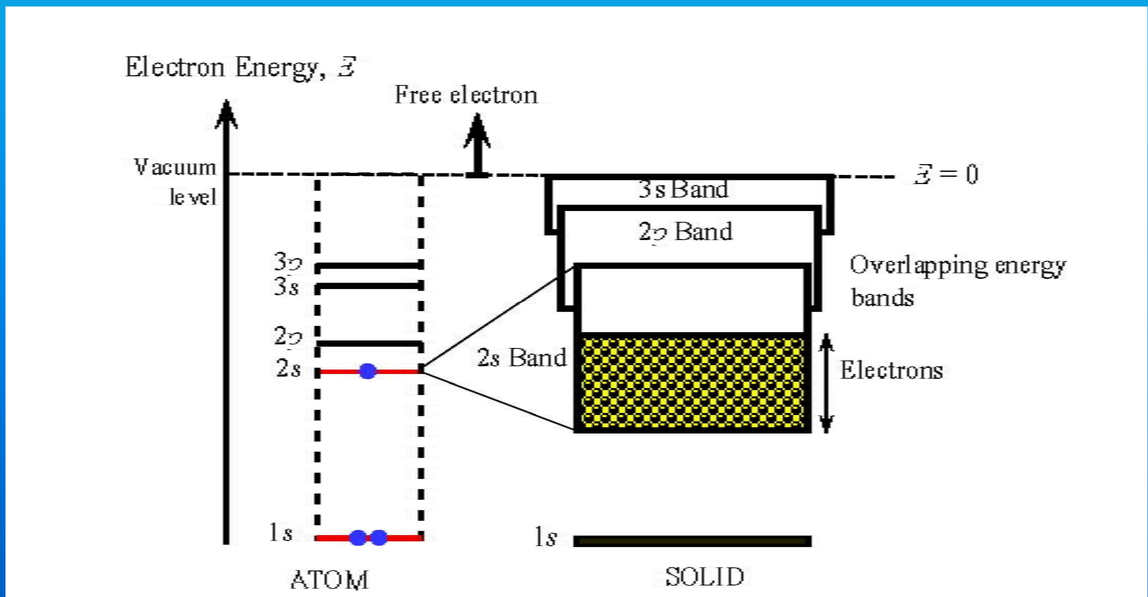
@ 12:00

Lecture 06

Semiconductor Science

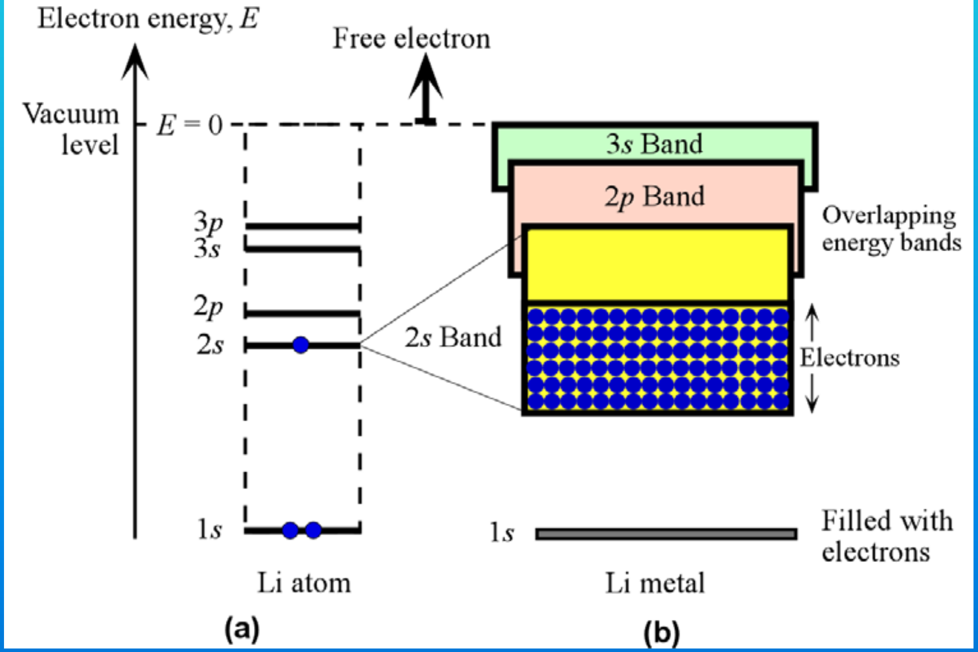
Energy Band Diagrams

- The electron energy in an atom/molecule is quantized and can have only certain discrete values.
- For example:
  - Li atom has two electrons in the $1s$ shell and one electron in the $2s$ subshell.
- If we bring together something like 10^{23} Li atoms to form a metal crystal the interatomic interactions results in electron energy bands. The $2s$ energy level splits into some 10^{23} closely spaced energy levels that effectively form an energy band - called the $2s$ band.



Energy Band Diagrams - Metals

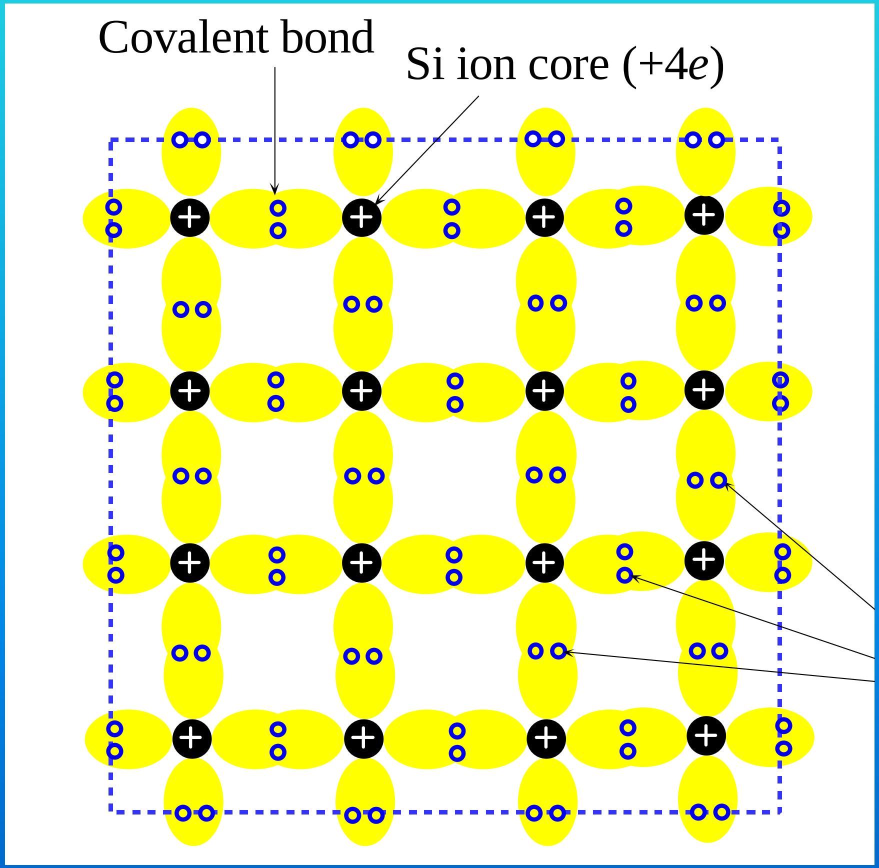
- Similarly, other higher energy levels also form bands.
- Various energy bands overlap to give a single band of energies that is only partially full of electrons.
- There are states with energies up to the vacuum level where the electron is free.



The $2s$ energy level in the Li atom is half full ($2s$ subshell needs 2 electrons), which means that the $2s$ band in the crystal will also be half full.
Metals characteristically have partially filled energy bands.

Energy Band Diagrams - Semiconductors

- The electron energies in a semiconductor crystal are different.
- A silicon crystal has each Si atom bonding to four neighbors with four valence electrons.



A simplified two dimensional view of a region of the Si crystal showing covalent bonds.

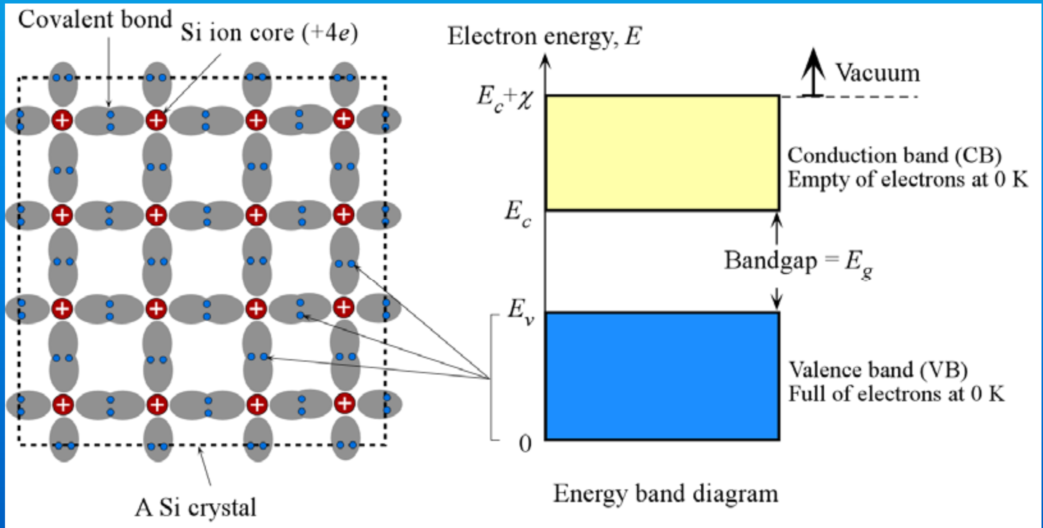
Energy Band Diagrams

The interactions between atoms and valence electrons result in:
→ the electron energy falling into two distinct energy bands
-- the valence band (VB) and conduction band (CB)
-- separated by an energy gap, bandgap (E_g) .

VB -- represents electron states (wavefunctions) in the crystal that correspond to bonds between the atoms. Electrons that occupy these VB are the valence electrons.
CB -- represents electron states (wavefunctions) ↔ higher energies than those in VB
Bandgap -- the forbidden electron energies.

(a) A simplified two dimensional view of a region of the Si crystal showing covalent bonds.

(b) The energy band diagram of electrons in the Si crystal at 0K. The bottom of the VB has been assigned a zero of energy.



(a) (b)

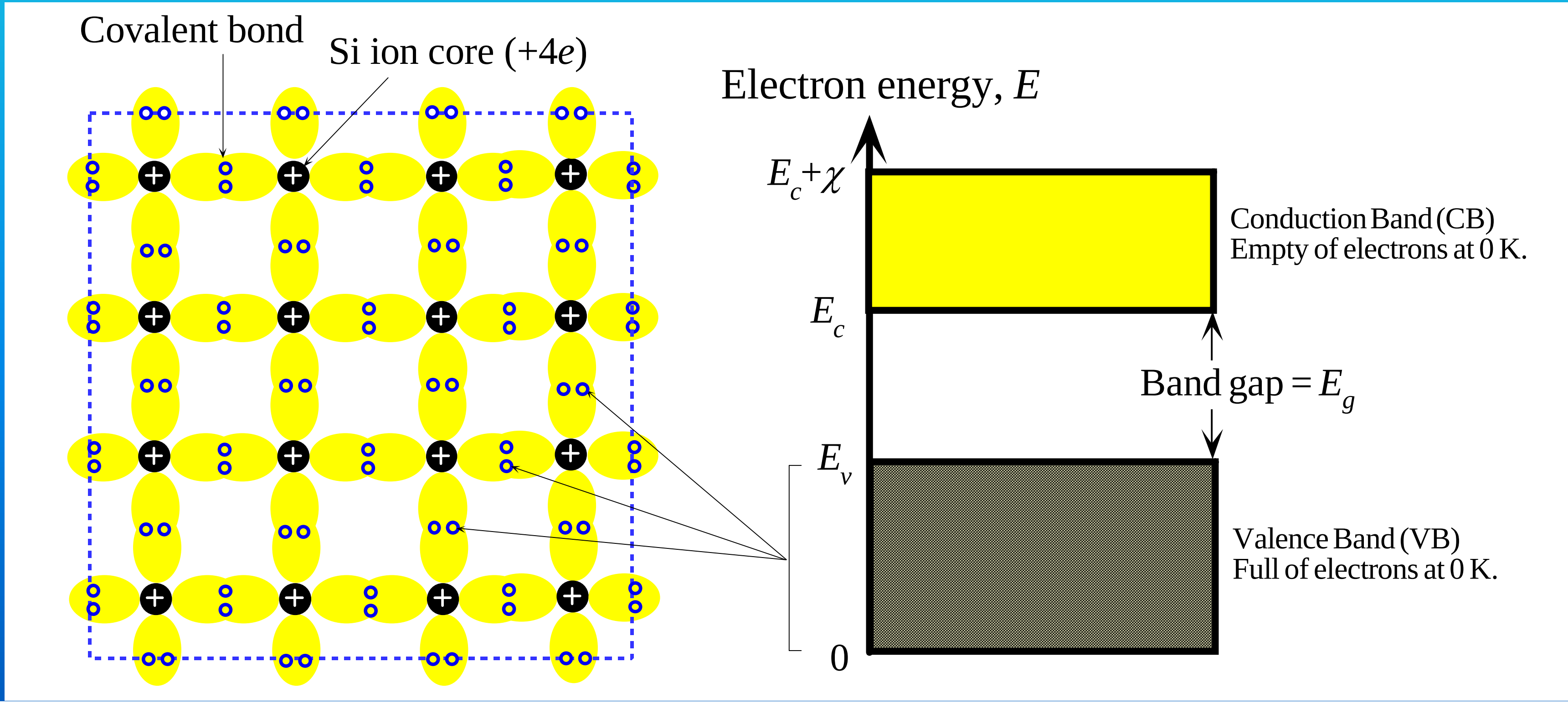
Energy Band Diagrams

The top of the VB ~ E_v, the bottom of the CB ~$E_c \Rightarrow E_g = E_c - E_v$.

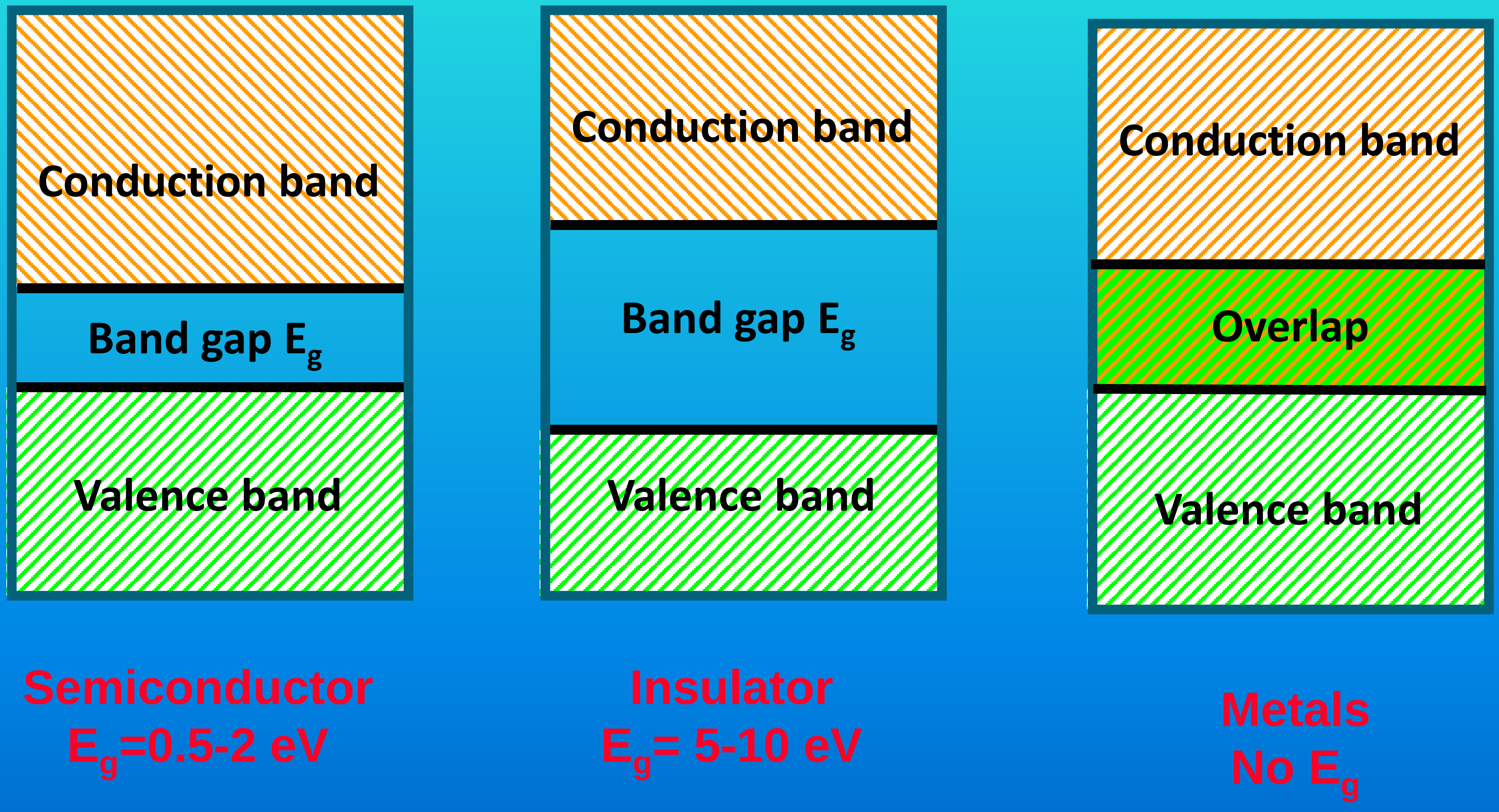
The width of the CB, χ is called the electron affinity.

In general, an electron in the CB can be treated as if it were free within the crystal by simply assigning an effective mass m_e^* to it.

The excitation of an electron from VB requires a minimum E_g.



Energy Band Structure at 0 K

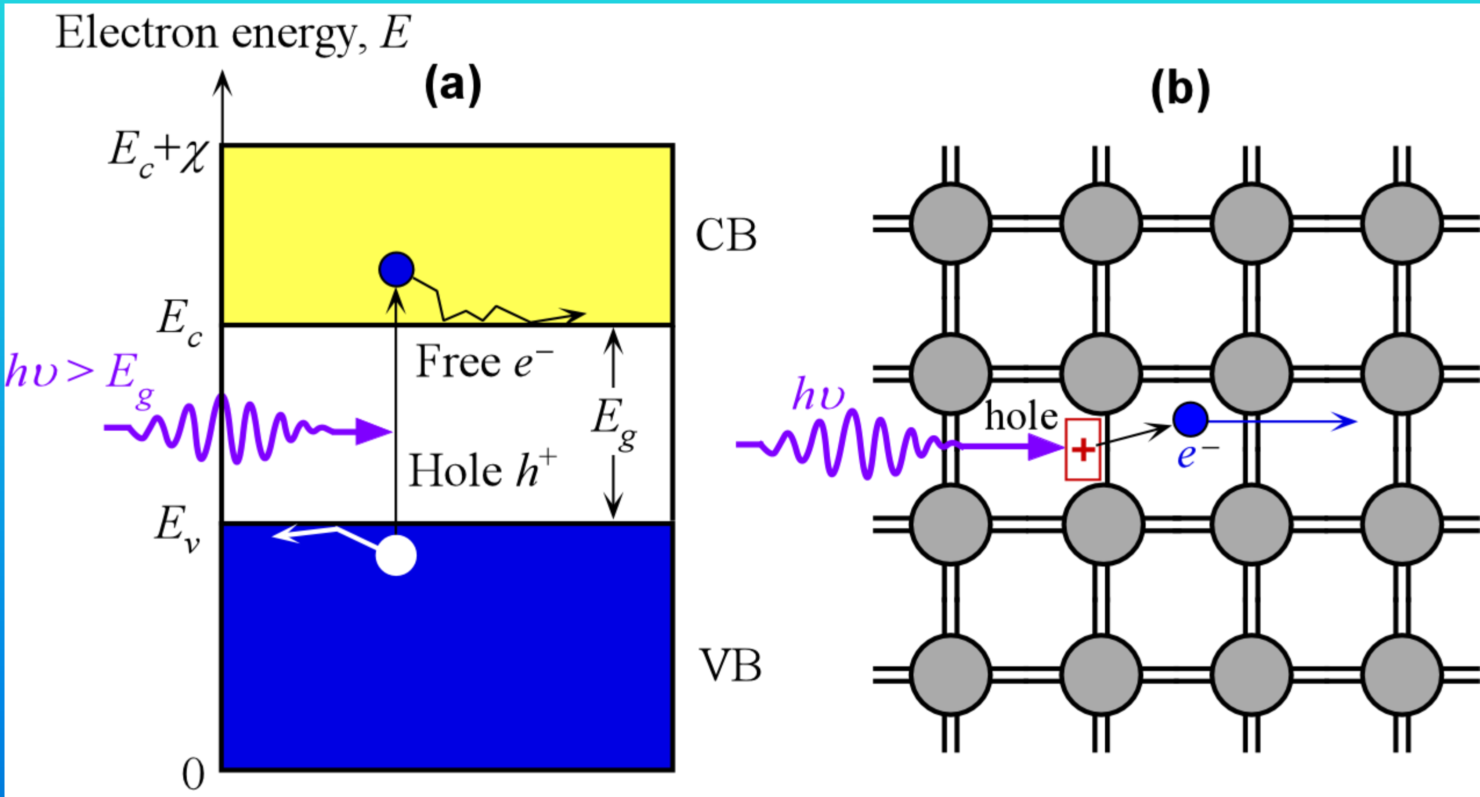


Electron-hole Pair Creation

An incident photon ($h\nu > E_g$) can excite an electron from VB to the CB

$\Rightarrow$ A free electron in the CB and a "hole" in the VB

-- an electron-hole pair (EHP) creation.



(a) A photon with an energy $h\nu$ greater than E_g can excite an electron from the VB to the CB. (b) Each line between Si-Si atoms represents a valence electron in a bond. When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created. The result is the photogeneration of an electron and a hole pair (EHP)

Electron-hole Pair Creation

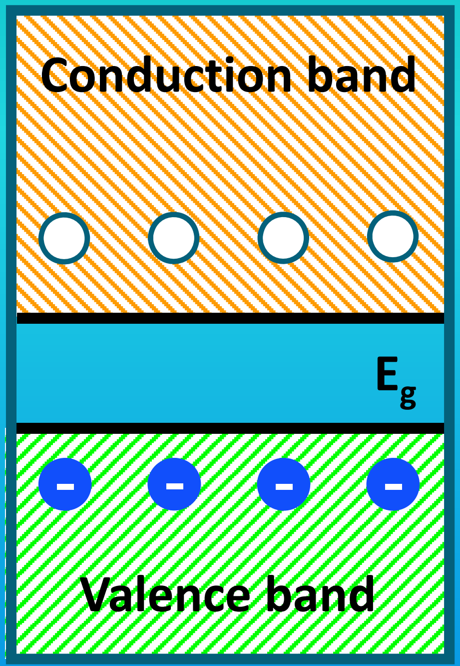
The empty electronic state, or the missing electron, in the bond is what we call a hole in the VB. The free electron, which is in the CB, can wander around the crystal and contribute to the electrical conduction when an electric field is applied.

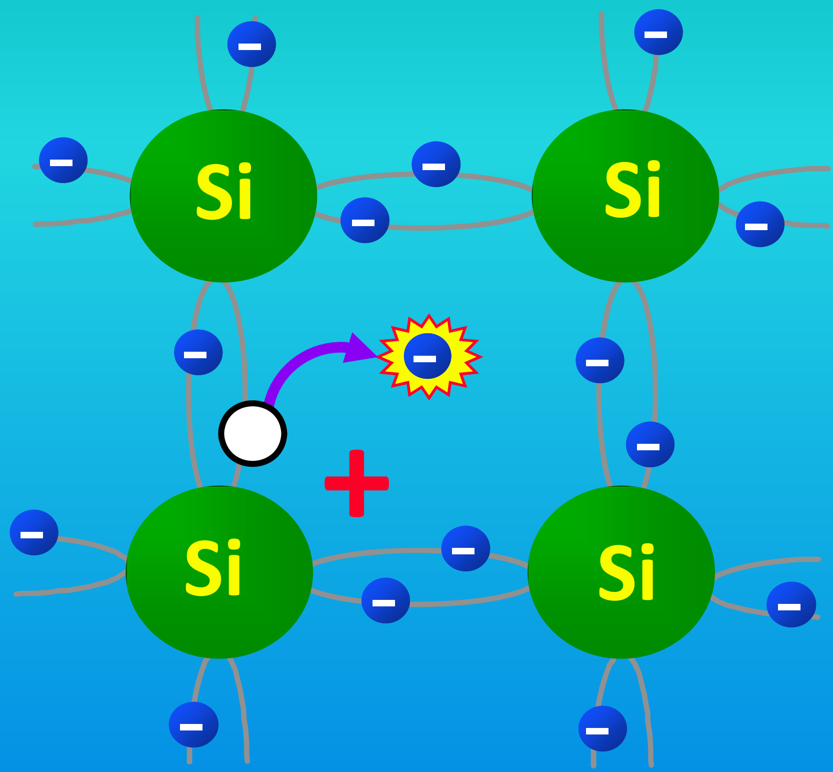
The region remaining around the hole in the VB is positively charged because a charge of *-e* has been removed from an otherwise neutral region of the crystal. This hole, denoted as h^+ can also wander around the crystal as if it was "free“.

This is because an electron in a neighbouring bond can "jump", *i.e. tunnel, into the hole to fill the vacant* electronic state at this site and thereby create a hole at its original position. This is effectively equivalent to the hole being displaced in the opposite direction.

Thus conduction in semiconductors occurs by both electrons and holes with charges *-e and +e* respectively and their own effective masses m_e^* *and* m_h^*.

Electron-hole Pair Creation





Semiconductor
E_g=0.5-2 eV

- Some electrons jump from the valence band to the conduction band. They are charge carriers because they can move from one atom to another.
- The empty state in the valence band is referred to as a “hole”.
- The holes have positive charge. They are also charge carriers.

Thermal Excitation

Although in this specific example a photon of energy $h\nu > E_g$ *creates an electron- hole* pair, other sources of energy can also lead to an electron-hole pair creation.

In the absence of radiation and temperatures above t = 0 K there is still an electron-hole generation process going on – thermal generation.

Due to thermal energy, the atoms in the crystal are constantly vibrating, which corresponds to the bonds between the Si atoms being periodically deformed with a distribution of energies.

Energetic vibrations can rupture bonds and thereby create electron and hole pairs by exciting electrons from the VB to the CB.

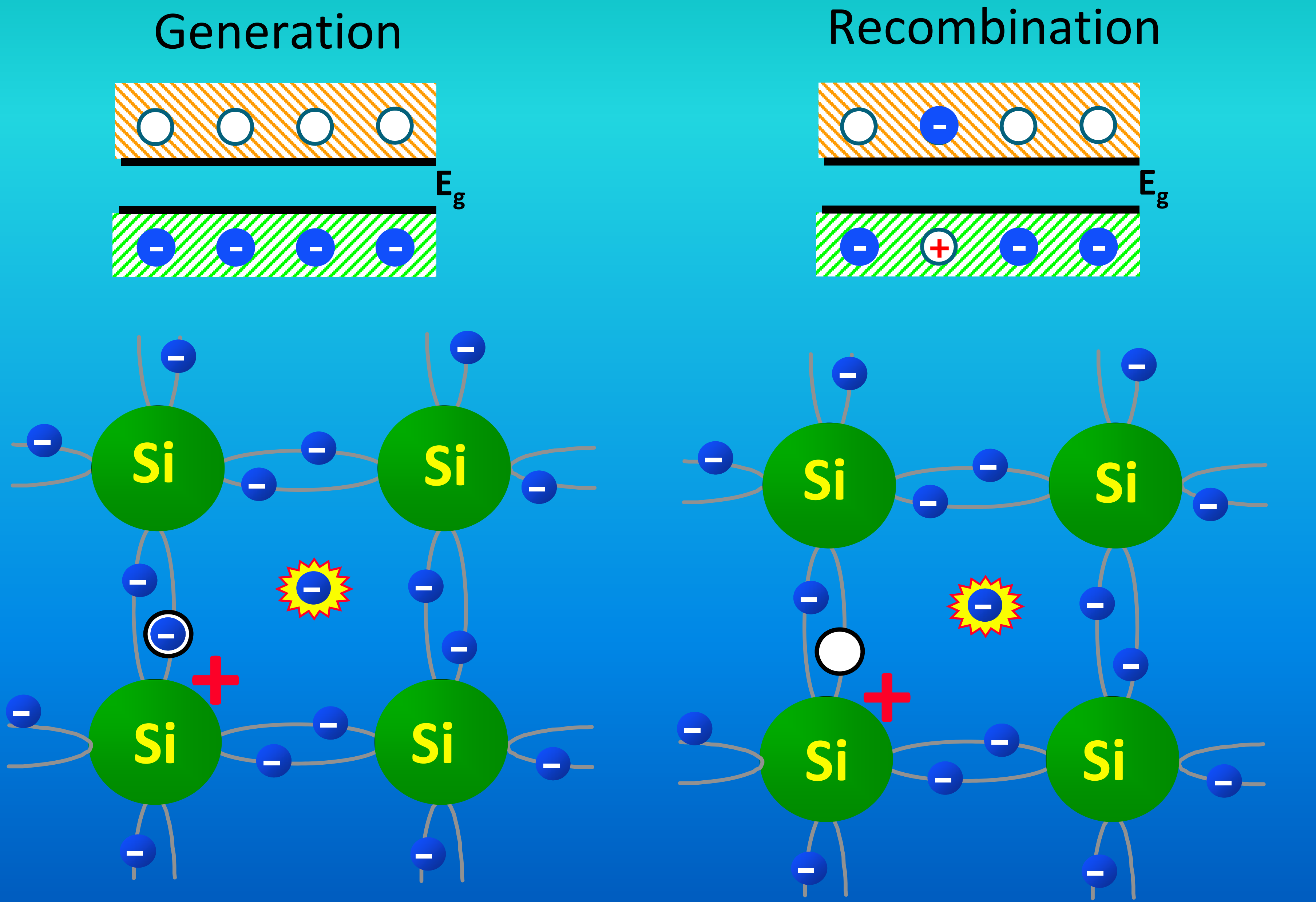
Recombination

When a wandering electron in the CB meets a hole in the VB, it has found an empty electronic state of lower energy and therefore occupies it. The electron falls from the CB to the VB to fill the hole. This process of filling a hole (trapping an electron) is called recombination.

The recombination results in the annihilation of an electron from the CB and a hole in the VB. The excess energy of the electron falling from CB to VB in certain semiconductors, such as GaAs and InP, is emitted as a photon. In Si and Ge, the excess energy is lost as lattice vibrations (heat).

In the steady state, the thermal generation rate is balanced by the recombination rate so that the electron concentration *n in the CB and hole concentration p in the VB remain constant;* both *n and p depend on the temperature.*

Generation and Recombination



Electron-hole Pair Creation - General

Conduction in semiconductors occurs by both electrons and holes with charges $-e$ and $+e$ and effective masses m_e* and m_h*.

An incident photon ($hv > E_g$) can excite an electron from VB to the CB
$\Rightarrow$ A free electron in the CB and a ”hole” in the VB

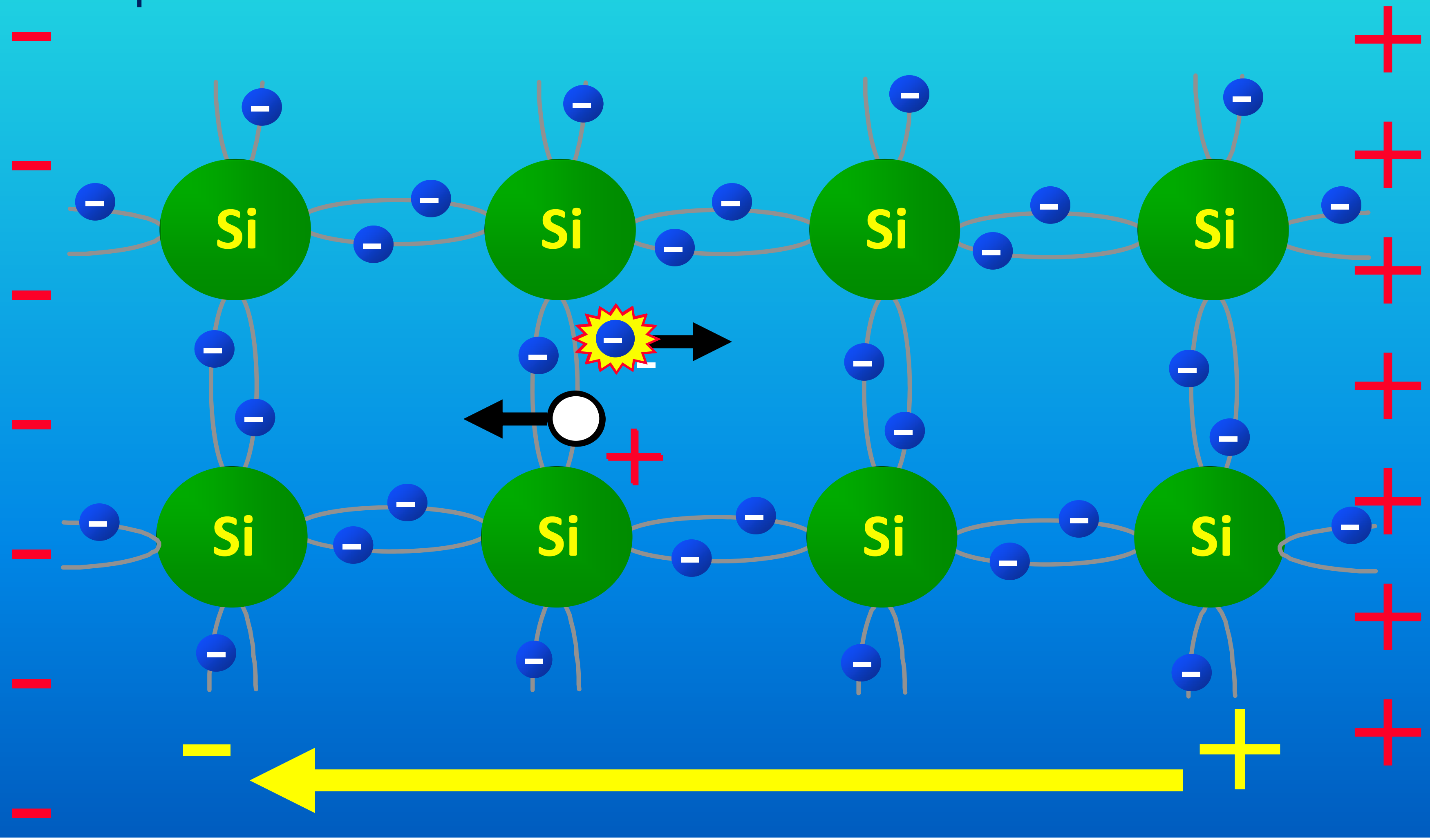
Thermal generation: An electron-hole generation process due to thermal energy (atom vibration).

Recombination: A wandering electron from CB $\rightarrow$ fills a hole in VB.

In steady state, the thermal generation rate $\leftrightarrow$ the recombination rate
$\rightarrow$ the electron concentration n in the CB and hole concentration p in the VB remain constant. (n, p ~T)

Electric Charge Carriers in Semiconductors

- In general, there will be electric current due to both electrons and holes
- Example: when there is an electric field in the semiconductor lattice



Semiconductor Statistic

There are two important concepts.

Density of states (DOS) $g(E)$ represents the number of electronic states (electron wavefunctions) in a band per unit energy per unit volume of the crystal.

The Fermi-Dirac function $f(E)$ - the probability of finding an electron in a quantum state with energy E.

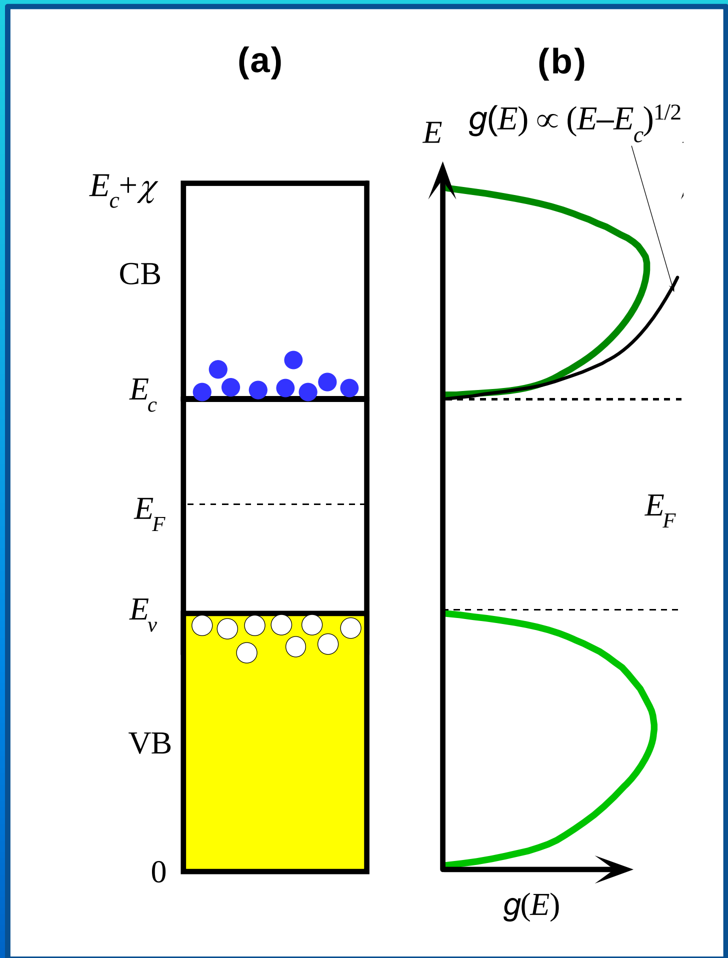
According to quantum mechanics for an electron confined within a three-dimensional potential energy well, the density of states increases with energy as

$$g(E) \propto \sqrt{E - E_c}$$

The energy $E - E_c$ is the electron energy from the bottom of the conduction band. The density of states function gives information only on available sates and not on their actual occupation.

Semiconductor Statistic

Density of states (DOS): The number of electronic states (electron wavefunctions) in a band per unit energy per unit volume of a crystal, $g(E)$.



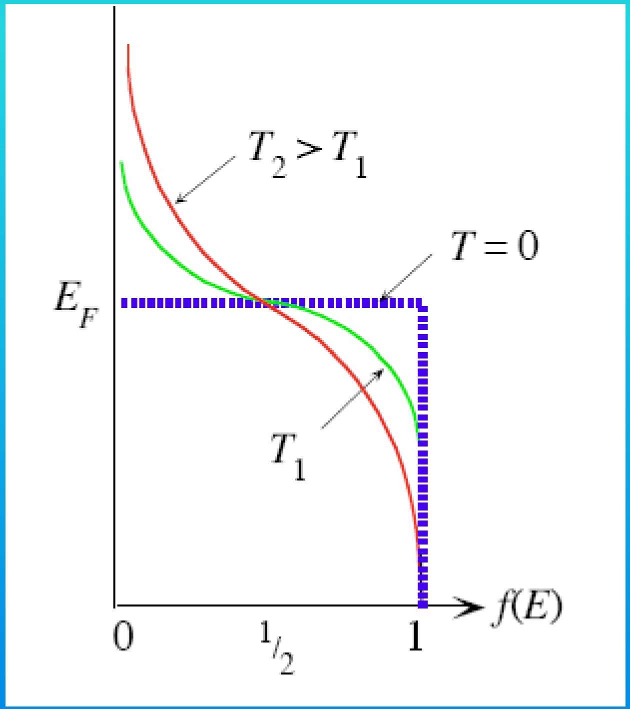
(a)Energy band diagram.
(b) Density of states (number of states per unit energy per unit volume).

Semiconductor Statistic

- The Fermi Dirac Function, f(E), is the probability of finding an electron in a quantum state with energy E. This function is a fundamental property of a collection of interacting electrons in thermal equilibrium

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

- Where k is the Boltzmann constant, T is the temperature in Kelvin, E_F is the Fermi energy



The Fermi energy is defined as the energy level where at T = 0 K, all states below it are filled.

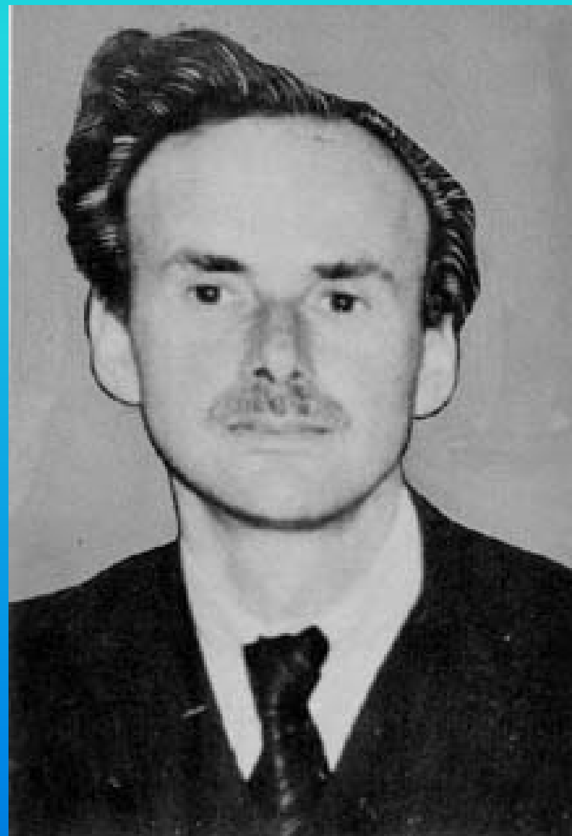
An equivalent definition is the Fermi energy is the level where $f(E)$ = ½ at some temperature T.

$f(E)$ = ½ means that ½ of the states are filled at this energy level.

Although at E_F the probability of electron occupancy is ½, there may be no states for an electron to occupy. What is important is the product of the density of states and the probability of occupation.

Semiconductor Statistic

Fermi Energy



Dirac Equation

Semiconductor Statistic

$f(E)$ is shown in (c) assuming E_F is located in the bandgap.

The product $g(E)\,f(E)$ is the actual number of electrons per unit energy per unit volume, $n_E(E)$, in the CB as shown (d).

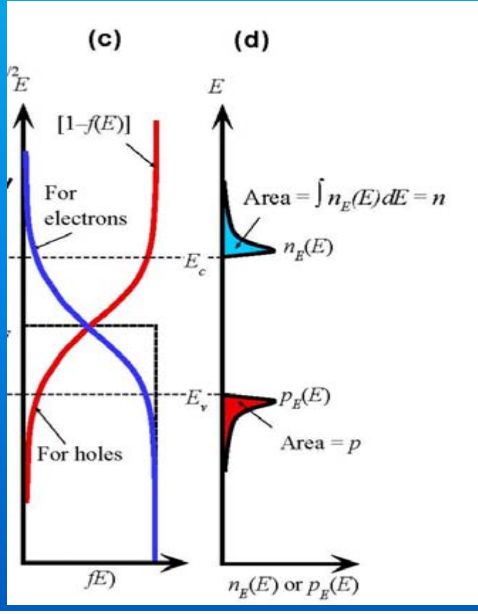
Non-degenerate semiconductors: When $(E_c - E_F) >> k_B T$ (E_F is at least a few $k_B T$ below E_c) $\Rightarrow f(E) \approx \exp[-(E_c - E_F)/k_B T]$

- in CB, the number of electrons << that of electronic states.

For non-degenerate semiconductors, the electron concentration

$$n \approx N_c \exp\left[-\frac{(E_c - E_F)}{k_B T}\right]$$

where $N_c = 2\left[2\pi m_e^* k_B T / h^2\right]^{3/2}$ is a constant called the effective density of states at the CB edge.



Semiconductor Statistic

hole concentration $p \approx N_v \exp\left[-\frac{(E_F - E_v)}{k_B T}\right]$

where $N_v = 2\left[2\pi m_h^* k_B T / h^2\right]^{3/2}$ is effective density of states at VB edge.

In an intrinsic semiconductors (pure crystal), $n = p$.

Thus $E_{Fi} = E_v + \frac{1}{2}E_g - \frac{1}{2}k_B T \ln\left(\frac{N_c}{N_v}\right)$

E_{Fi} - Fermi level in the intrinsic semiconductor (crystal)

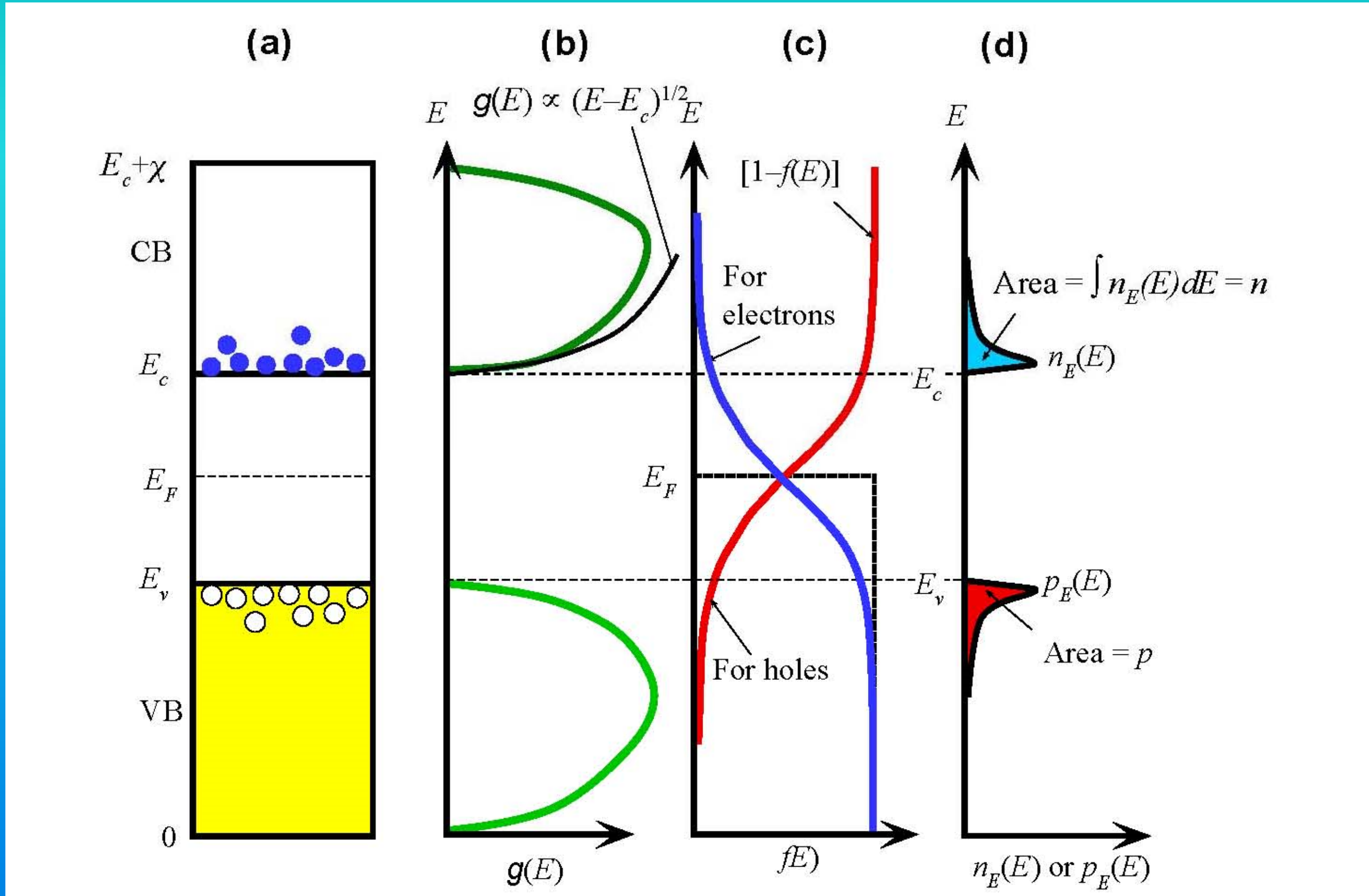
The mass action law is given by,

$$np = N_c N_v \exp\left[-\frac{E_g}{k_B T}\right] = n_i^2$$

in which $E_g = E_c - E_v$ is the bandgap energy

where n_i is called the intrinsic concentration.

Semiconductor Statistic



(a) Energy band diagram. (b) Density of states (number of states per unit energy per unit volume). (c) Fermi-Dirac probability function (probability of occupancy of a state). (d) The product of $g(E)$ *and* $f(E)$ is the energy density of electrons in the CB (number of electrons per unit energy per unit volume). The area under $n_E(E)$ *vs.* E *is* the electron concentration.

Extrinsic Semiconductor

Extrinsic semiconductors :

The concentration of carriers of one polarity >> the other type

---- by introducing small amounts of impurities

***n*-type semiconductors** :

Electron (majority) concentration >>Hole (minority) concentration

---- by adding pentavalent impurities (arsenic, As) in Si.

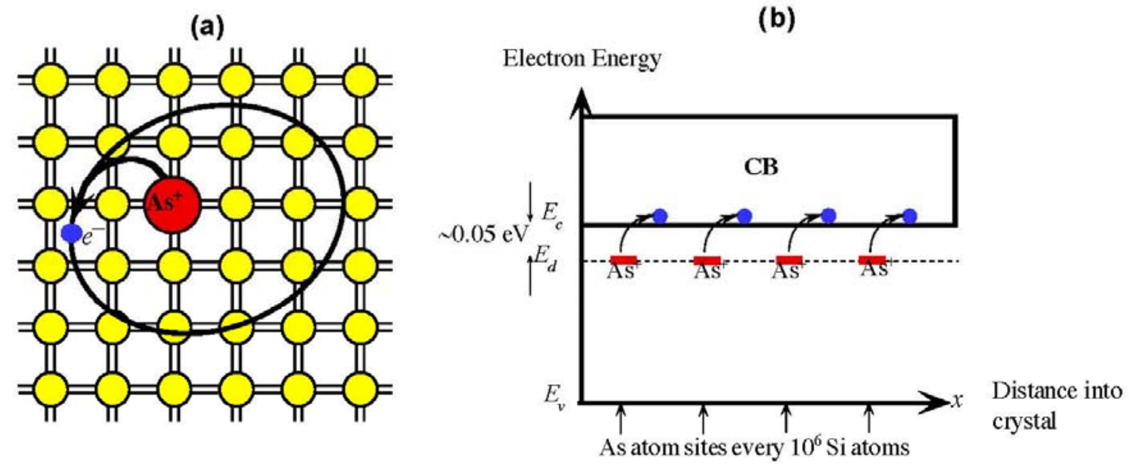
***p*-type semiconductors** :

Hole (majority) concentration>>Electron (minority) concentration

---- by adding trivalent impurities (boron, B) in Si.

n-type Extrinsic Semiconductor

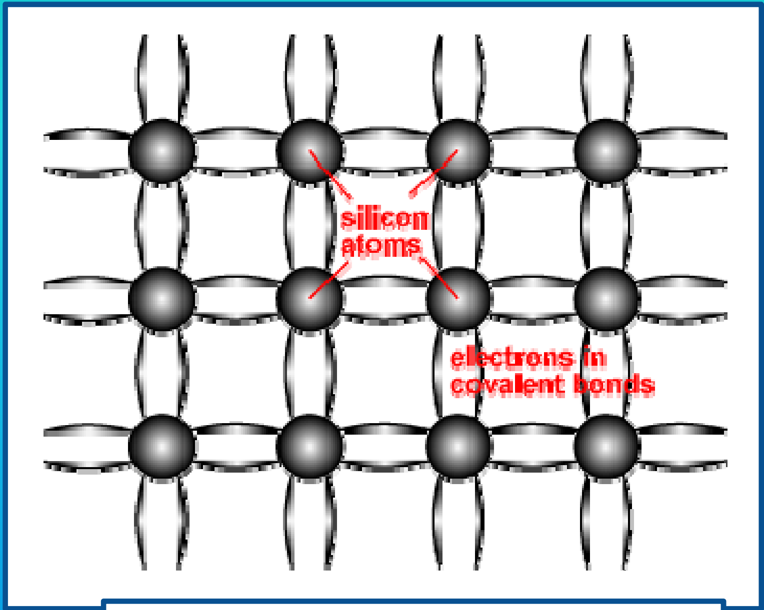
- Extrinsic semiconductors with excess electrons - n-type semiconductors
- Arsenic added to silicon - when an As atom bonds with four Si atoms, it has one electron left unbounded. This fifth electron cannot find a bond to go into, so it is left orbiting around the As atom.
- Arsenic is called a donor because it donates electrons to the system.
- For $N_d >> n_i$, at room temperature, the electron concentration inside the conduction band will be nearly equal to N_d such that $N_d = n$
- Number of holes: $p = n_i^2 / N_d$

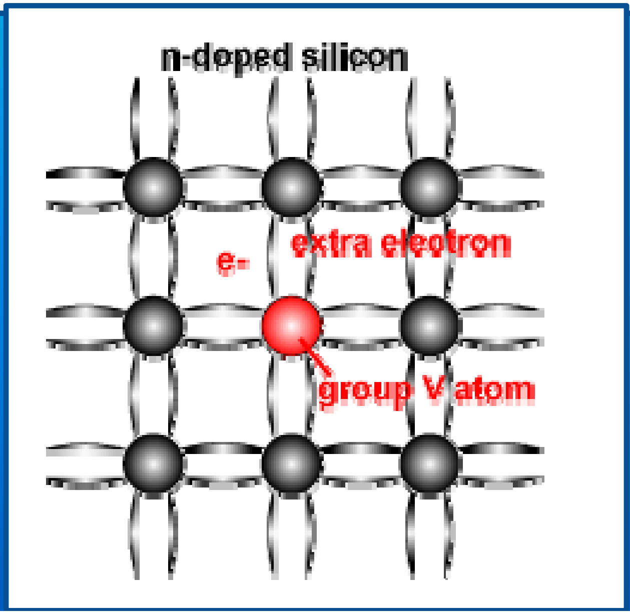


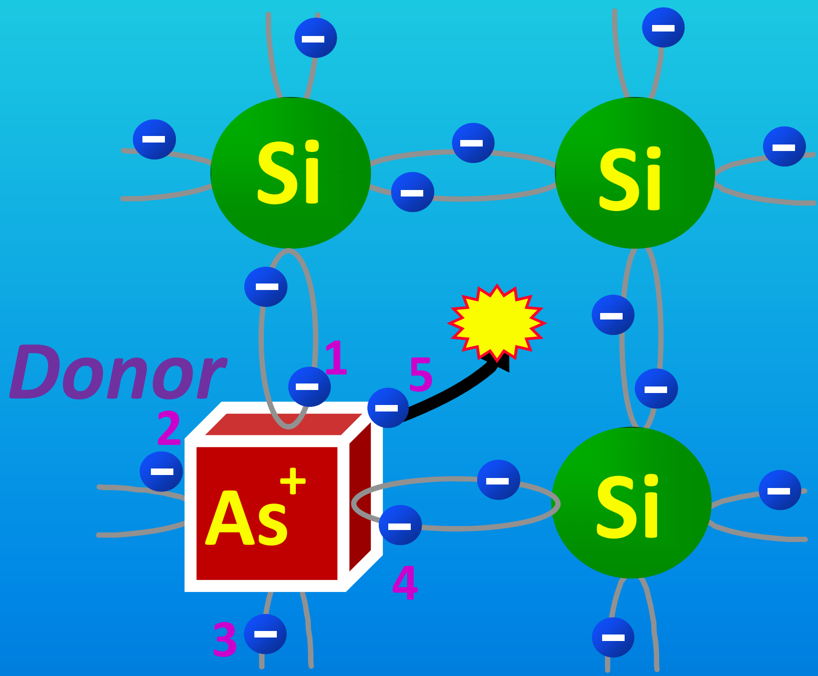
(a) The four valence electrons of As allow it to bond just like Si but the fifth electron is left orbiting the As site. The energy required to release to free fifth-electron into the CB is very small.

(b) Energy band diagram for an *n*-type Si doped with 1 ppm As. There are donor energy levels just below E_c around As^+ sites.

n-type Extrinsic Semiconductor





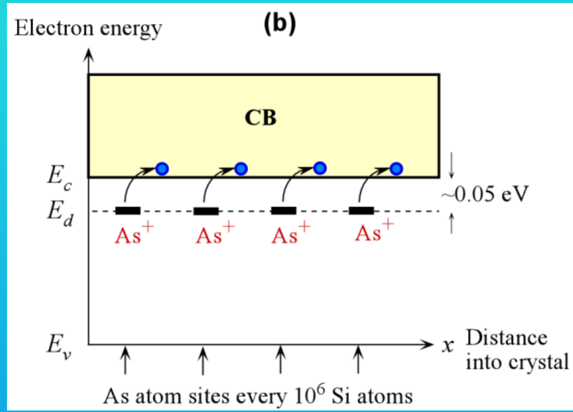


n-type Extrinsic Semiconductor

The energy required to free this fifth electron away from the As site ~0.05 eV ≈≈ the thermal energy at room temperature ($k_B T$ = 0.025 eV)

--- the electron can be readily freed by thermal vibrations of Si lattice

→ the CB ⇒ free electrons, as in (b).



The As atom donating an electron is called a donor impurity.

The electron energy E_d around the donor atom ~ < E_c - 0.05 eV . It means - thermal excitation by lattice vibrations at room temperature is sufficient to ionize the As atom

If N_d is the donor atom concentration, provided that $N_d >> n_i$,

at room temperature, the electron concentration in CB $n \approx N_d$ & the hole concentration in VB $p \approx n_i^2/N_d$. ($k_B T \Rightarrow n$).

The hole concentration is less than the intrinsic concentration because a few of the large number of electrons in the CB recombine with holes in the VB to maintain

$$np = n_i^2.$$

where $n_i^2 = np$ is called the intrinsic concentration.

n-type Extrinsic Semiconductor

The conductivity of a semiconductor depends on both electrons and holes and is given by,

$$\sigma = en\mu_e + ep\mu_h$$

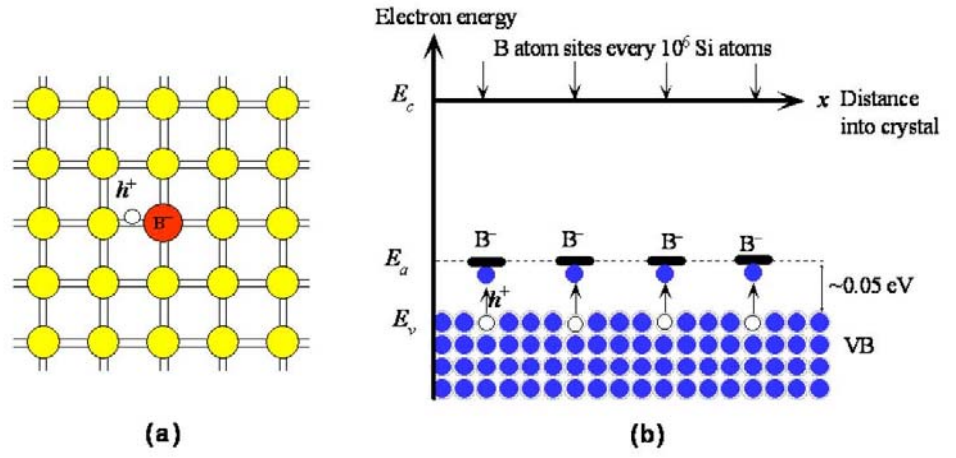
where μ_e & μ_h are the drift mobilities of the electron and holes respectively.

For an *n*-type semiconductor it becomes,

$$\sigma = eN_d\mu_e + e\left(\frac{n_i^2}{N_d}\right)\mu_h \approx eN_d\mu_e$$

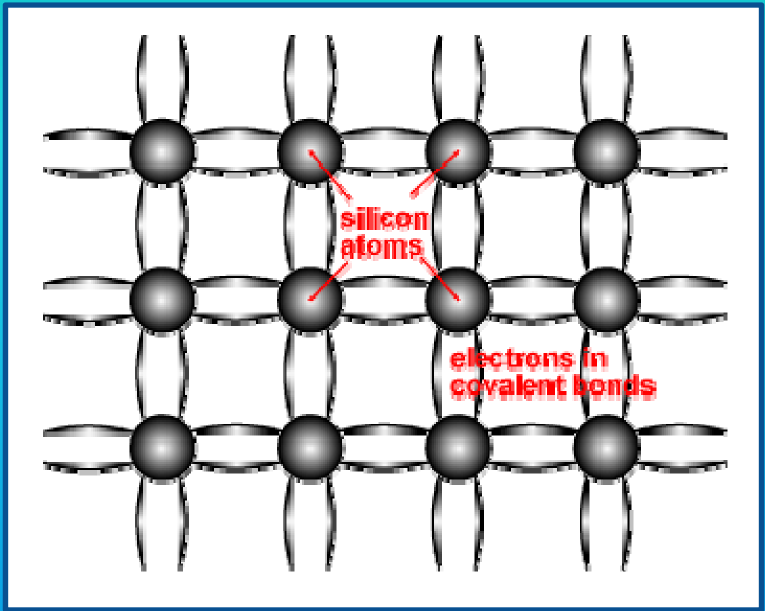
p-type Extrinsic Semiconductor

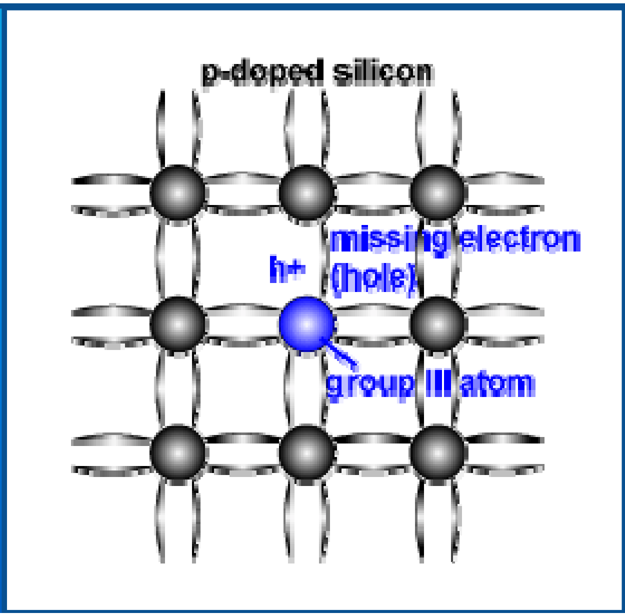
- Extrinsic semiconductors with less electrons - p-type semiconductors
- Adding Boron (+3) metal which has one fewer electron and yields an increased hole per doped atom.
- Boron is called an acceptor.
- For $N_a >> n_i$, at room temperature, the hole concentration inside the valence band will be nearly equal to N_a such that $N_a = p$
- Electron carrier concentration is determined by the mass action law as: $n = n_i^2 / N_a$

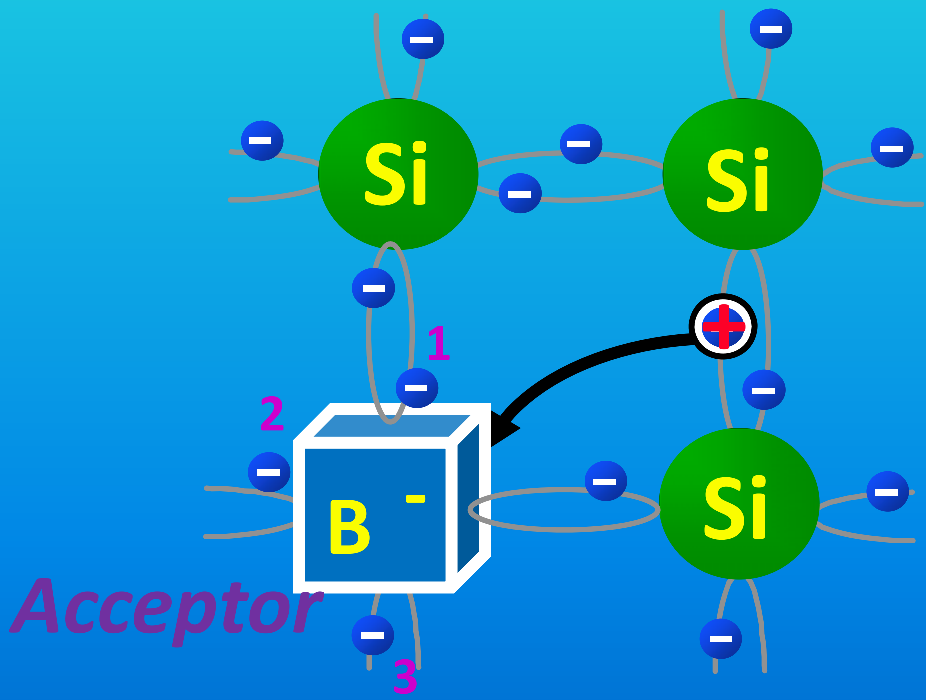


(a) Boron doped Si crystal. B has only three valence electrons. When it substitutes for a Si atom one of its bonds has an electron missing and therefore a hole. (b) Energy band diagram for a *p*-type Si doped with 1 ppm B. There are acceptor energy levels just above E_v around B− sites. These acceptor levels accept electrons from the VB and therefore create holes in the VB.

p-type Extrinsic Semiconductor

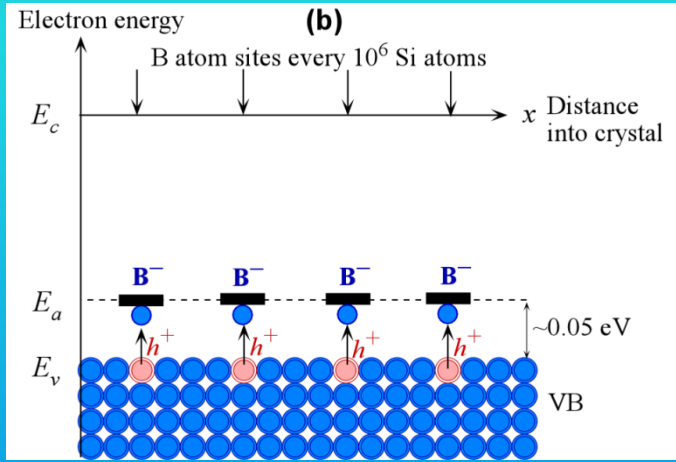






p-type Extrinsic Semiconductor

The energy required to free this hole away from the B site (or accept an electron) ~ 0.05 eV ≈≈ the thermal energy (~ $k_B T$ = 0.025 eV).
--- the hole can be readily freed by thermal vibrations of the Si lattice
$\rightarrow$ the VB $\Rightarrow$ free holes.



The B atom accepting an electron is called a acceptor impurity.
The electron energy E_a around the acceptor atom ~ < E_v - 0.05 eV .

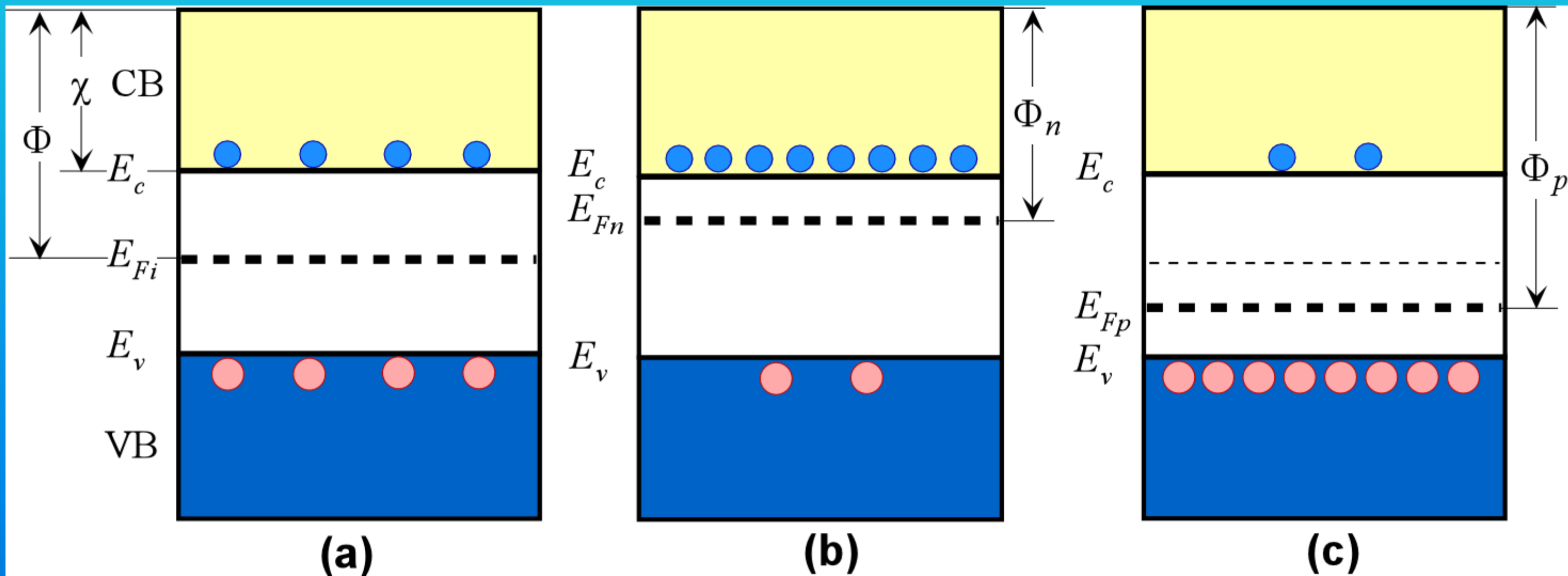
If N_a is the acceptor atom concentration, provided that $N_a >> n_i$,
at room temperature, the hole concentration in VB $p \approx N_a$
and the electron concentration in CB $n \approx n_i^2/N_a$.

Similarly, for a p-type semiconductor, the conductivity is given by:

$$\sigma \approx e N_a \mu_h$$

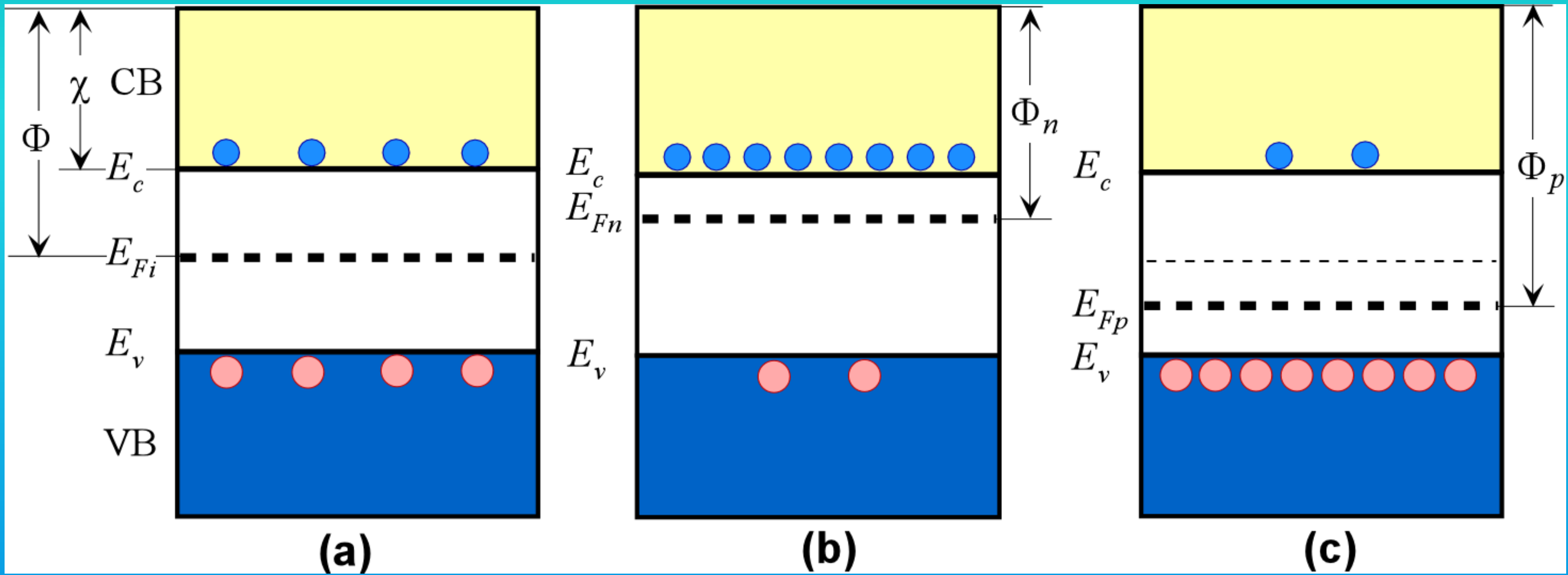
Simplified Band Diagram for Semiconductors

- Notice that the Fermi level changes as a function of doping
- Notice also that carrier concentration (h / e) also changes as a function of doping
- N-type: majority carriers are electrons and minority carriers are holes
- P-type: majority carriers are holes and minority carriers are electrons
- Mass action law valid: $n_{no}p_{no}=n_i^2$ where *no* is the doped equilibrium carrier concentr.



Intrinsic, *i*-Si	Extrinsic *n*-type	Extrinsic *p*-type
$n = p = n_i$	$n = N_d$	$p = N_a$
	$p = n_i^2/N_d$	$n = n_i^2/N_a$
	$np = n_i^2$	$np = n_i^2$

Simplified Band Diagram for Semiconductors

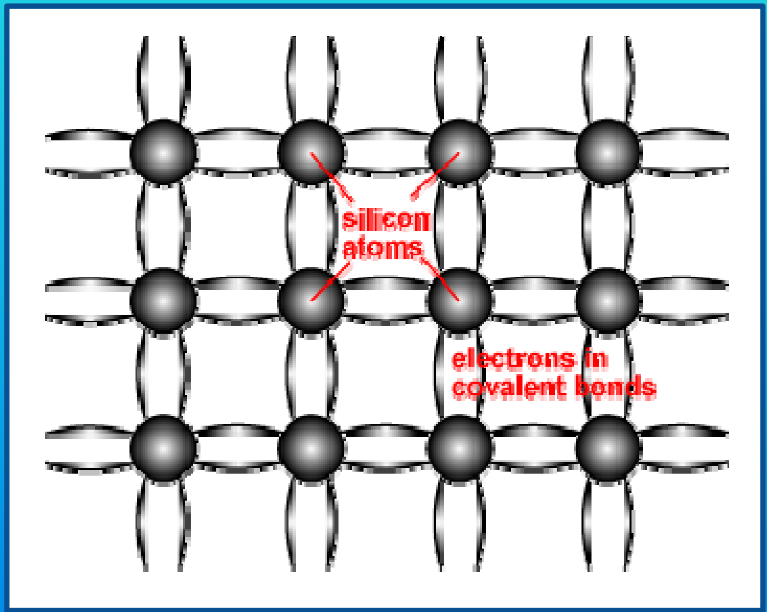


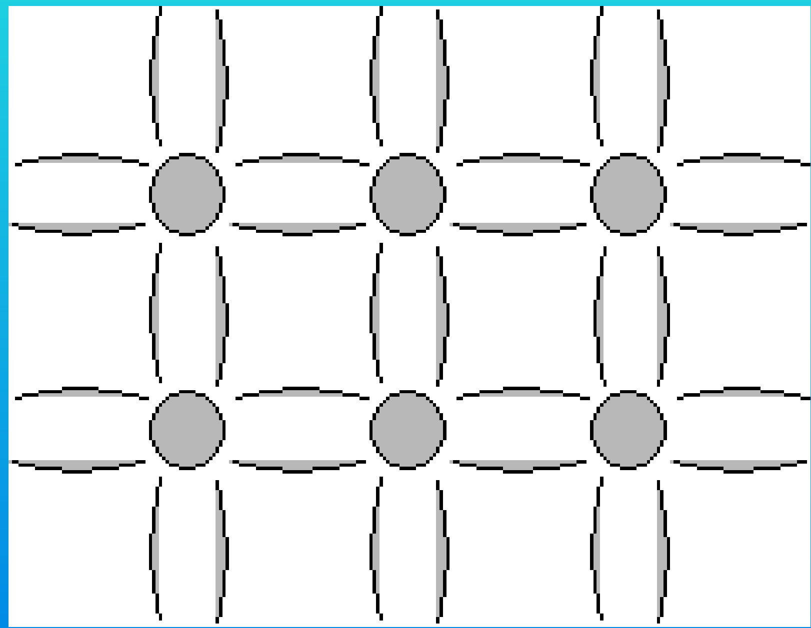
Energy band diagrams for (a) intrinsic (b) *n*-type and (c) *p*-type semiconductors.

In all cases, $np = n_i^2$. Note that donor and acceptor energy levels are not shown.

CB = Conduction band, VB = Valence band, E_c = CB edge, E_v = VB edge, E_{Fi} = Fermi level in intrinsic semiconductor, E_{Fn} = Fermi level in *n*-type semiconductor, E_{Fp} = Fermi level in *p*-type semiconductor. χ is the electron affinity. Φ, Φ_n and Φ_p are the work functions for the intrinsic, *n*-type and *p*-type semiconductors

Extrinsic Semiconductors





Extrinsic Semiconductor - Compensation Doping

Compensation doping :

The doping of a semiconductor with both donors and acceptors to control the properties.

Provides precise control of carrier concentrations

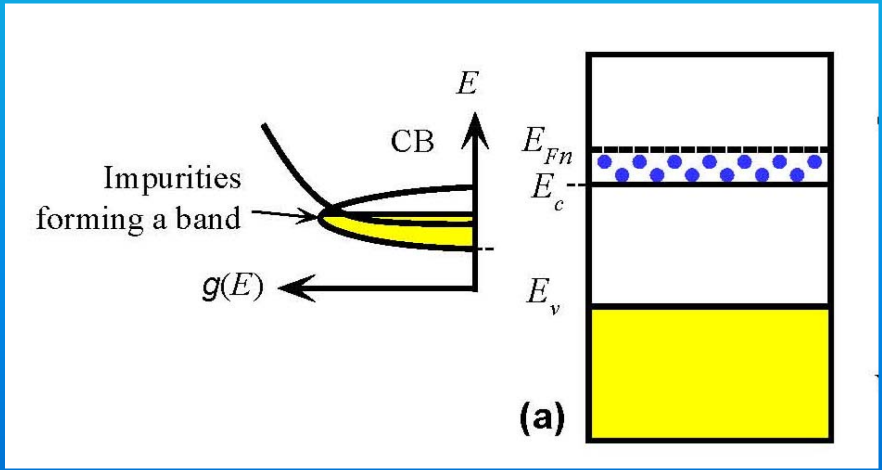
$$n = N_d - N_a$$

Non-degenerate semiconductors :

The number of states N_c in the CB >> the number of electrons n

Si typically n_i ~ 10^{10} cm^{-3}

Degenerate *n-type* semiconductor. Large number of donors form a band that overlaps the CB.



Degenerate semiconductors :

The number of electrons n > the number of states N_c in the CB by excessively doped with donors, typically 10^{19} ~ 10^{20} cm^{-3}

Extrinsic Semiconductor - Compensation Doping

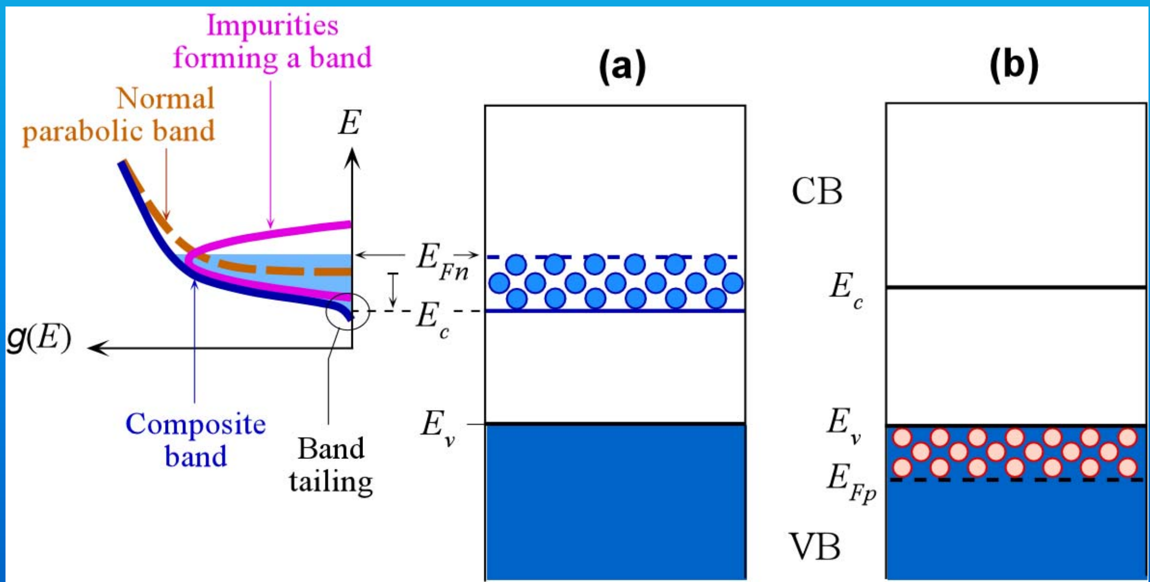
Non-degenerate semiconductors :

The number of states N_c in the CB >> the number of electrons n

Si typically n_i ~ 10^{10} cm^{-3}

Degenerate semiconductors :

The number of electrons n > the number of states N_c in the CB by excessively doped with donors, typically 10^{19} ~ 10^{20} cm^{-3}



(a) Degenerate *n*-type semiconductor. Large number of donors form a band that overlaps the CB. E_c is pushed down and E_{Fn} is within the CB.

(b) (b) Degenerate *p*-type semiconductor.

Extrinsic Semiconductor - Compensation Doping

Compensation doping describes the doping of a semiconductor with both donors and acceptors to control the properties.

Example: A *p*-type semiconductor doped with N_a acceptors can be converted to an *n*-type semiconductor by simply adding donors until the concentration N_d exceeds N_a.

The effect of donors compensates for the effect of acceptors.

The electron concentration $n = N_d - N_a > n_i$

When both acceptors and donors are present, electrons from donors recombine with the holes from the acceptors so that the mass action law $np = n_i^2$ is obeyed.

We cannot simultaneously increase the electron and hole concentrations because that leads to an increase in the recombination rate which returns the electron and hole concentrations to values that satisfy $np = n_i^2$.

$$N_d > N_a$$

$$n = N_d - N_a$$
$$p = n_i^2/(N_d - N_a)$$

Extrinsic Semiconductor - Compensation Doping

Summary of Compensation Doping

More donors than acceptors $N_d - N_a >> n_i$

$$n = N_d - N_a$$

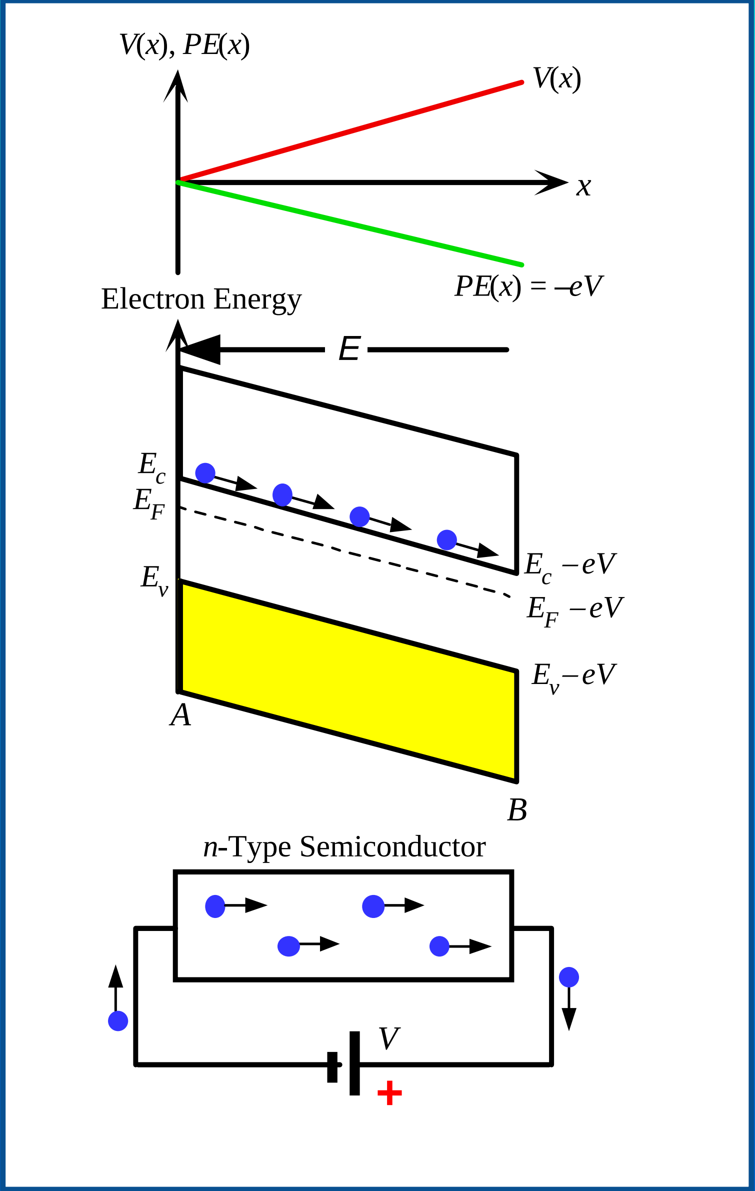
$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a}$$

More acceptors than donors $N_a - N_d >> n_i$

$$p = N_a - N_d$$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_a - N_d}$$

Energy Band Diagram in an Applied Field

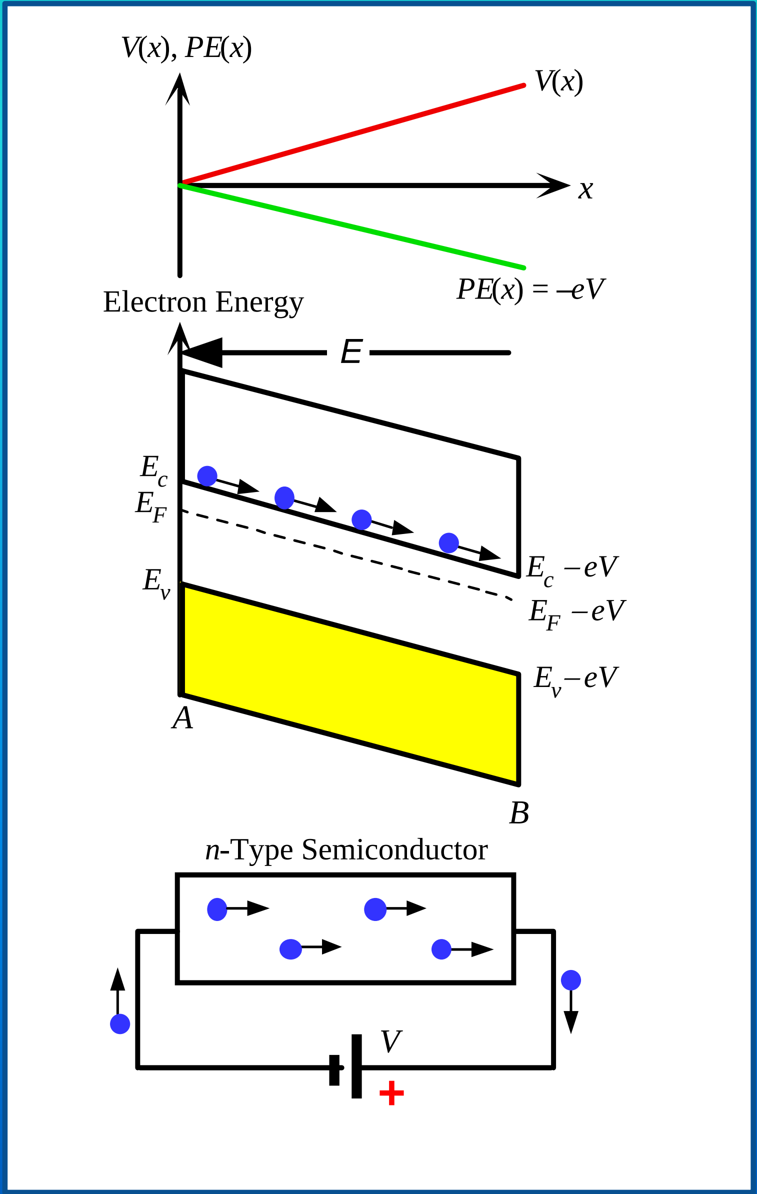


Consider the energy band diagram for an n-type semiconductor that is connected to a voltage supply of V volts and is carrying a current.

The Fermi level E_F is above that for the intrinsic case (E_{Fi}). That is, closer to E_c than E_v.

The applied voltage drops uniformly along the semiconductor so that the electrons in the semiconductor now also have an imposed electrostatic potential energy, which decreases towards the positive terminal.

Energy Band Diagram in an Applied Field



E_F must be uniform across the system only for a semiconductor in thermal equilibrium and in the dark.

When a potential is applied, E_F follows the electrostatic potential energy behavior.

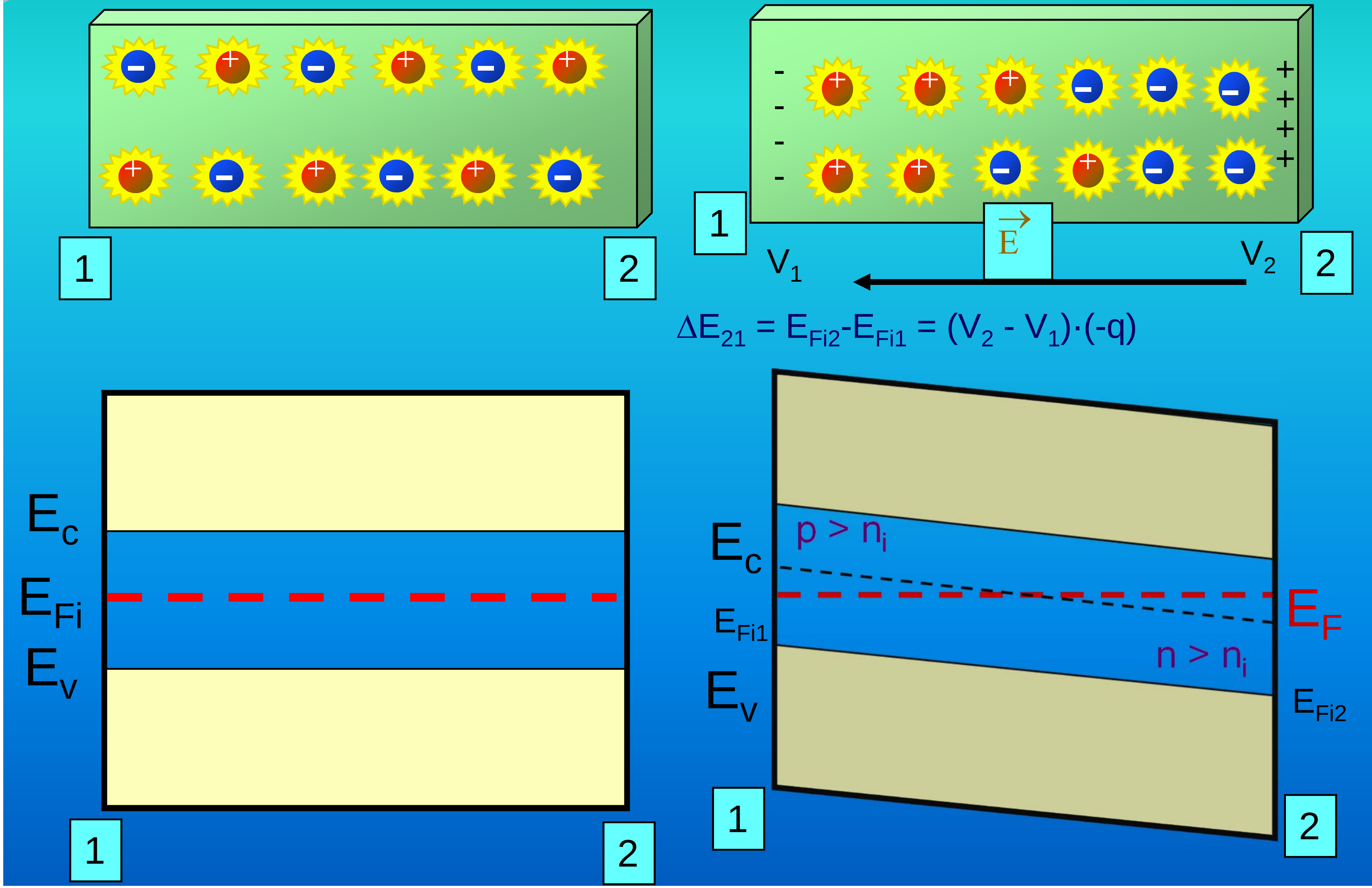
E_F(A) − E_F(B) is just eV.

Electron concentration in the semiconductor is uniform so:

$\Rightarrow E_c - E_F$ must be constant from one end to the other, and the CB, VB, and E_F all bend by the same amount.

Energy band diagram of an *n-type semiconductor* connected to a voltage supply of *V* volts. The whole energy diagram tilts because the electron now has an electrostatic potential energy as well

Energy Band Diagram in an Applied Field



Direct & Indirect Bandgap Semiconductors: E-k Diagrams

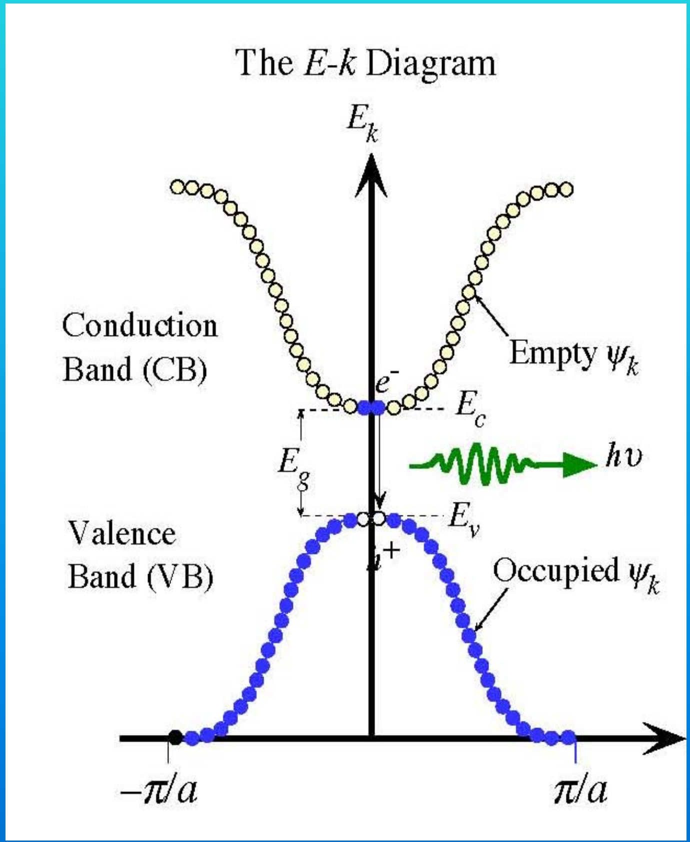
We know from quantum mechanics that when the electron is within an infinite potential energy well of spatial width L, its energy is quantized and given by

$$E_n = \frac{(\hbar k_n)^2}{2m_e}$$

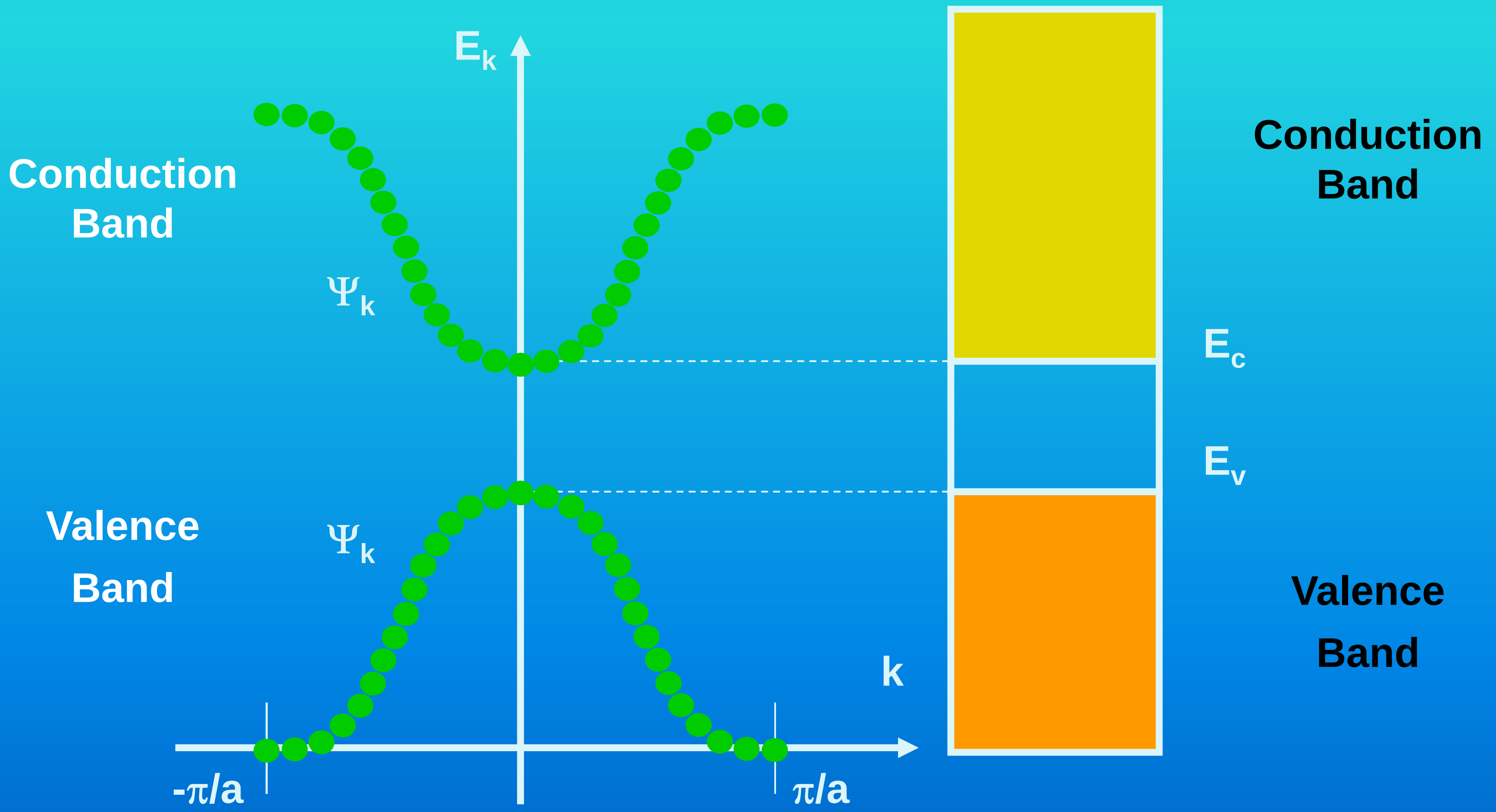
Where "h-bar" is ***Planck constant*** and m_e is the electron mass and the wavevector k_n is a quantum number determined by

$$k_n = \frac{n\pi}{L} \quad \text{where } n = 1,2,3...$$

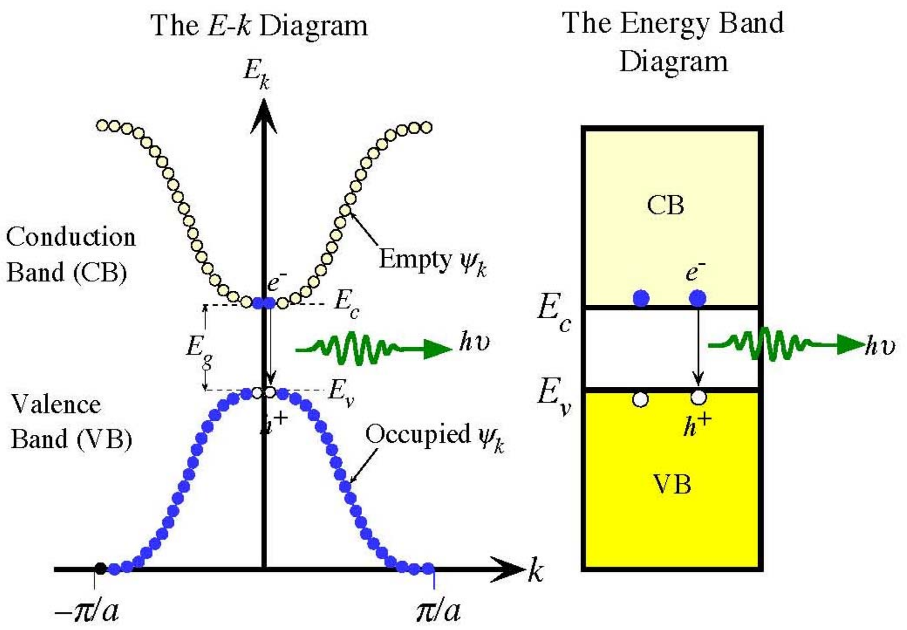
The energy increases parabolically with the wavevector k_n .



Direct & Indirect Bandgap Semiconductors



Direct & Indirect Bandgap Semiconductors



The *E*-*k* diagram of a direct bandgap semiconductor such as GaAs. The *E*-*k* curve consists of many discrete points with each point corresponding to a possible state, wavefunction $\psi_k(x)$, that is allowed to exist in the crystal. The points are so close that we normally draw the *E*-*k* relationship as a continuous curve. In the energy range E_v to E_c there are no points ($\psi_k(x)$ solutions).

- Momentum, p, in the crystal is $\hbar k$
- External forces:

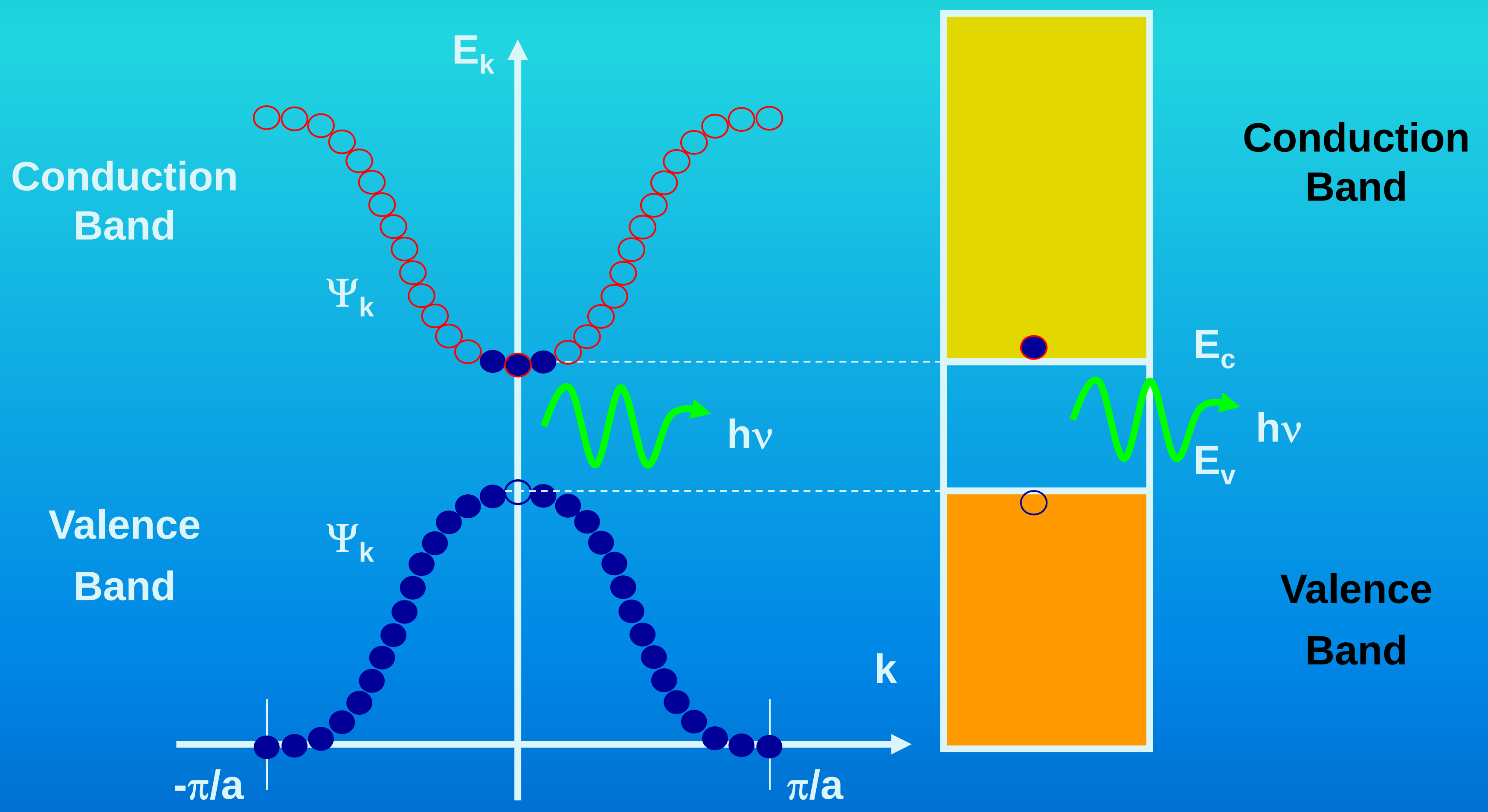
$$F = \frac{dp}{dt} = \frac{d(\hbar k)}{dt}$$

E - k diagram is an energy vs. crystal momentum plot

- The states $\Psi_k(x)$ in the lower *E-k curve constitute the wave-functions for the valence electrons and thus* correspond to the states in the VB.

- Those in the upper *E - k curve, on the* other hand, correspond to the states in the CB, since they have higher energies. All the valence electrons at 0 K therefore fill the states (particular k_n values) in the lower *E - k diagram.*

Direct & Indirect Bandgap Semiconductors



Direct & Indirect Bandgap Semiconductors

Direct bandgap semiconductor (GaAs):

When an electron and hole recombine (> 0 K),

the bottom of CB $\rightarrow$ the top of VB without any change in its k

---- The minimum of CB is directly above the maximum of VB.

Indirect bandgap semiconductor (Si):

An electron at the bottom of CB cannot recombine directly with a hole at the top of VB, with $k_{cb} \rightarrow k_{vb}$

---- The minimum of CB is not directly above the maximum of VB

The recombination process occurs via a recombination center (defects or impurities) at E_r within the bandgap.

---- The electron is first captured by the defect at E_r

--- the hole.

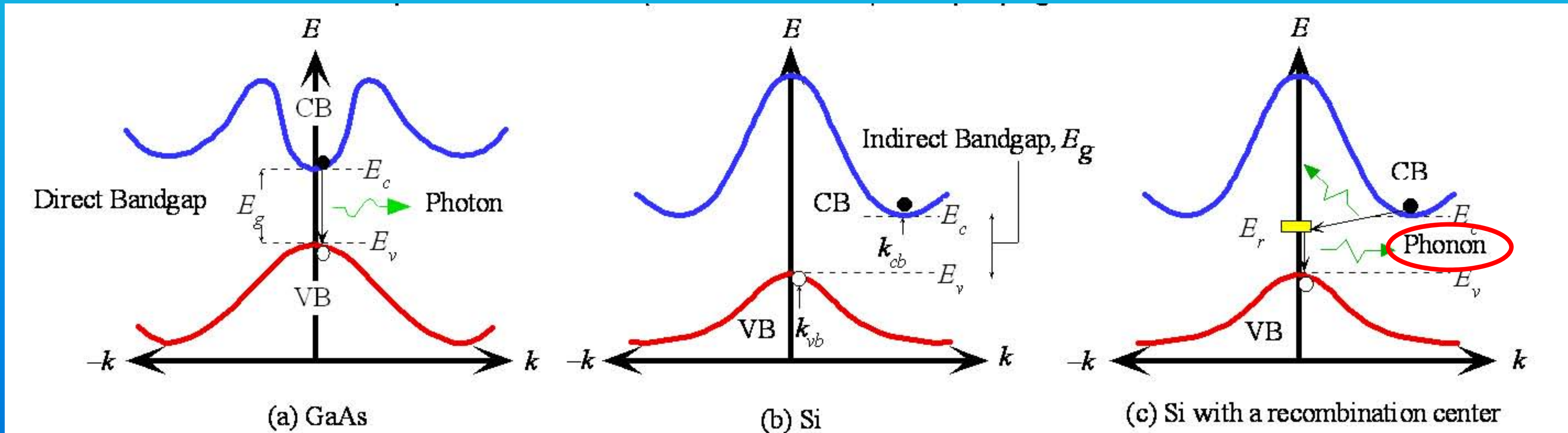
Direct & Indirect Bandgap Semiconductors

Direct Bandgap

- Base of the conduction band (CB) is matched to the max height of the valence band (VB)
- Recombination through the emission of a photon (Light!!!!!)

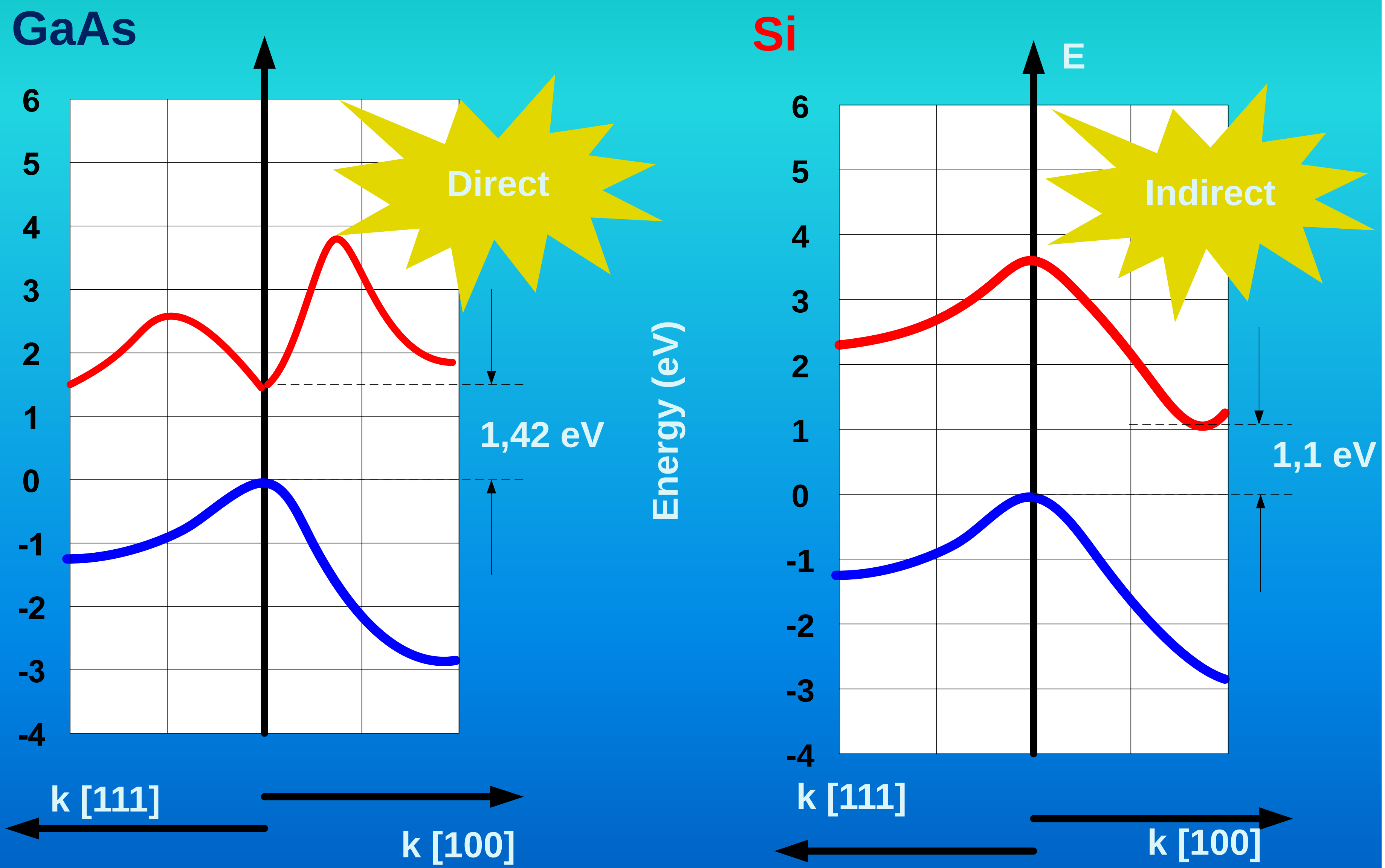
Indirect Bandgap

- Direct recombination would require a momentum change (not allowed)
- Recombination centers (lattice defects) are required to recombine CB to VB bands
- The result is a phonon emission (lattice vibration) that propagates across the lattice



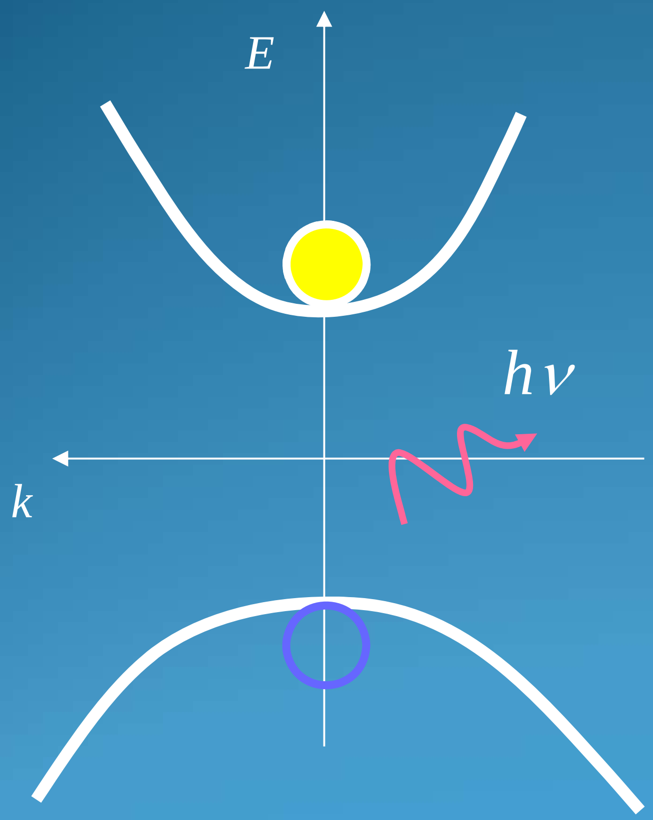
(a) In GaAs the minimum of the CB is directly above the maximum of the VB. GaAs is therefore a direct bandgap semiconductor. (b) In Si, the minimum of the CB is displaced from the maximum of the VB and Si is an indirect bandgap semiconductor. (c) Recombination of an electron and a hole in Si involves a recombination center .

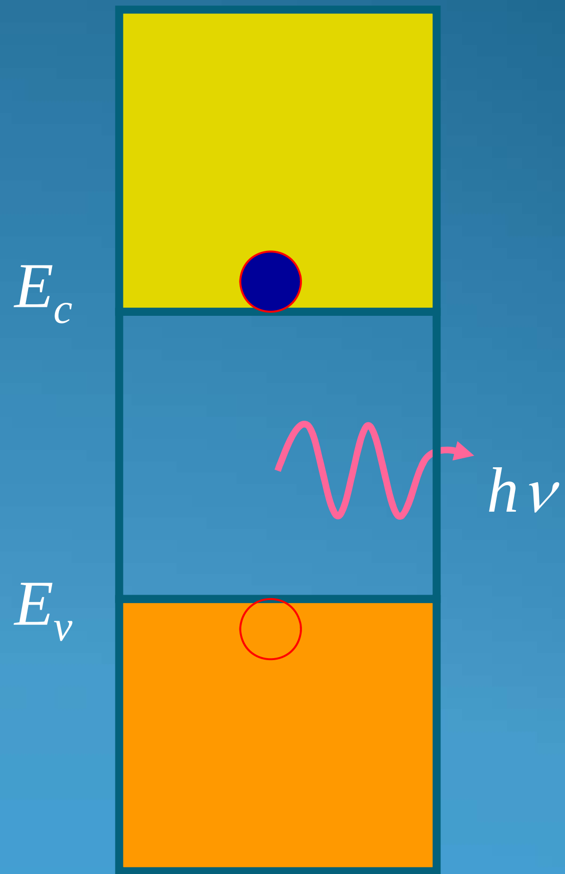
Direct & Indirect Bandgap Semiconductors



Direct Bandgap Semiconductors

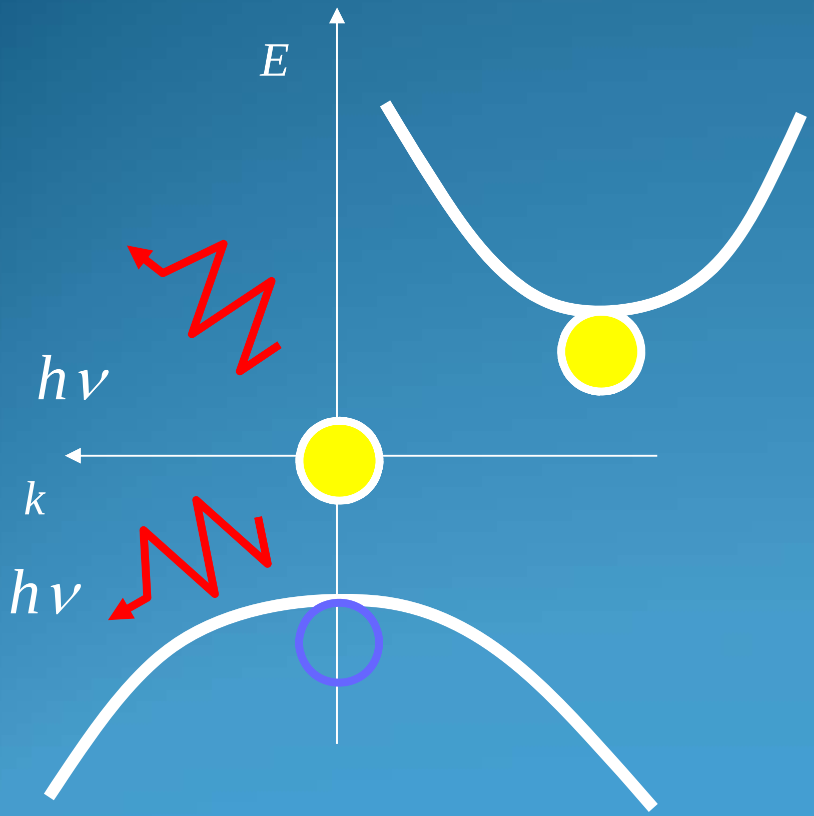
GaAs

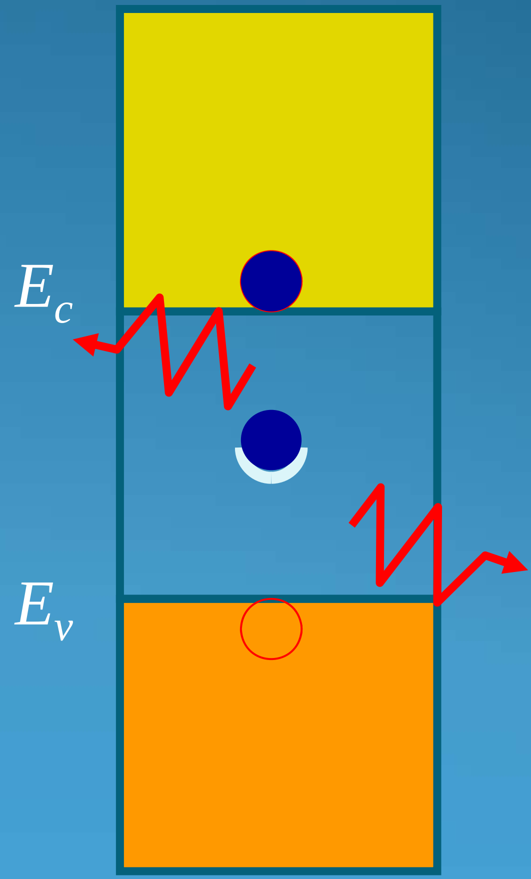




Indirect Bandgap Semiconductors

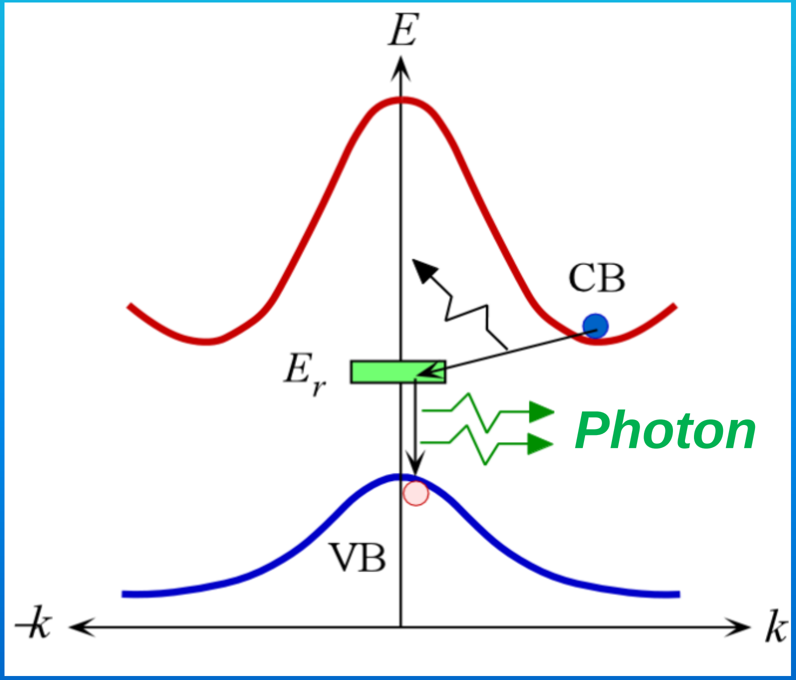
Si

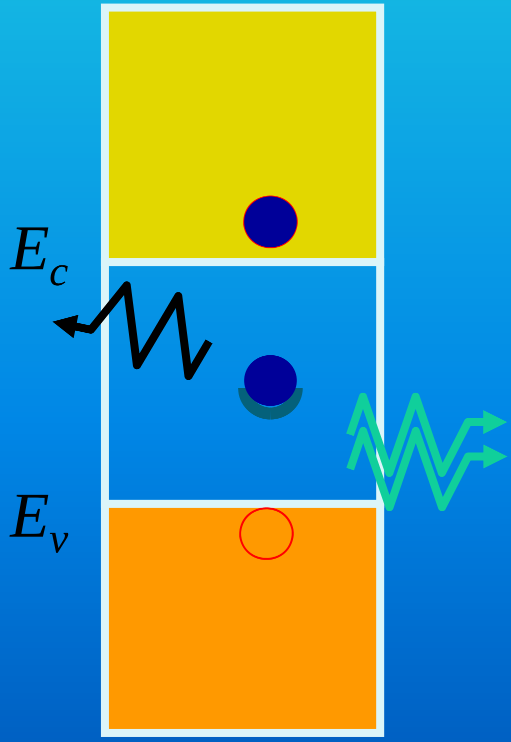




Indirect Bandgap Semiconductor

- In some indirect bandgap semiconductors such as GaP, however, the recombination of the electron with a hole at certain recombination centers results in photon emission.
- The recombination centers at E_r are generated by the purposeful addition of nitrogen (N) impurities to GaP, written as GaP:N. The electron transition from E_r *to* E_v *involves photon emission in the green.*





GaP with N recombination center (GaP:N)