

# Optics, Spring 2018

## Exercise 1, 18.1.2018

### 1. Complex number representation

- Show that multiplying a complex number by  $\pm i$  is equivalent to phase shift of  $\pm 90^\circ$ .
- Two waves of the same amplitude, frequency and speed are overlapping in some region of space:

$$\psi(y, t) = A \cos(ky + \omega t) + A \cos(ky - \omega t + \pi)$$

Show using complex exponentials that:

$$\psi(y, t) = -2A \sin ky \sin \omega t$$

### 2. Maxwell's equations and waves

- Derive the the free space wave equation for electric and magnetic fields from Maxwell's equations.
- How does the situation change in linear dielectric medium?
- What is the physical basis of the assumption that in typical dielectrics  $\mu_r = 1$ .

### 3. Three dimensional waves

Harmonic plane wave propagates in linear dielectric with homogenous  $\epsilon$  and  $\mu_r = 1$ . Derive the expression for magnetic field (amplitude, phase and polarization), if the complex electric field is linearly polarized in x-direction:

$$\mathbf{E} = E_0 e^{-i(\omega t - kz)} \hat{x}$$

### 4. Spherical waves

Show that the function

$$\bar{\psi}(r, t) = \frac{A}{r} e^{i(\omega t - kr)} \hat{r}$$

is a solution of the three-dimensional wave equation.