tonics 17 Risto Montonen

$N_a = 10^{18}\ cm^{-3}$, $N_d = 5 \cdot 10^{15}\ cm^{-3}$,

and $n_i = 10^{10}\ cm^{-3}$ (in Si)

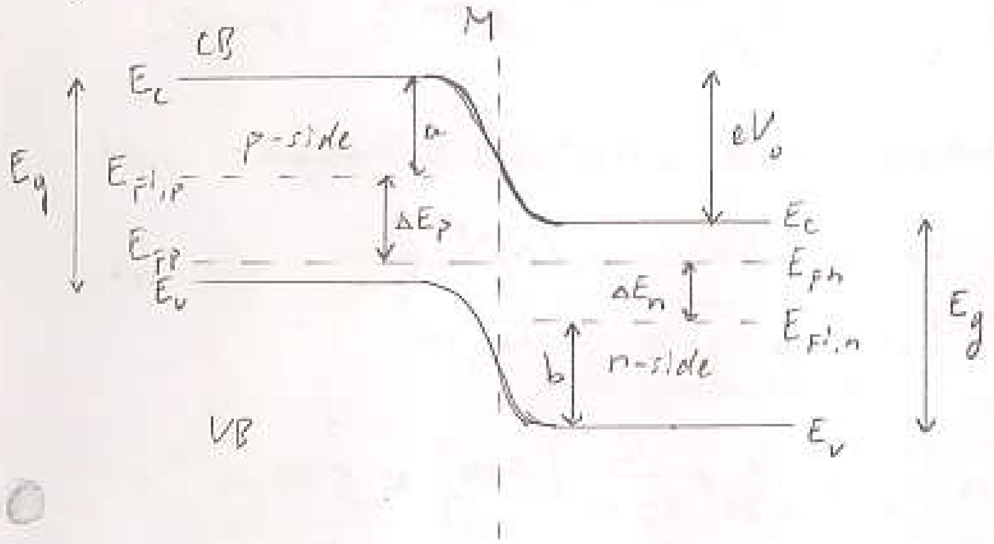
a) Fermi levels in the n region:

$$\Delta E_n = E_{Fn} - E_{Fi} = k_B T \ln\left(\frac{N_d}{n_i}\right) \overset{\text{at room temp.}}{\approx} \underline{\underline{0.340\ eV}}$$ above intrinsic level

in the p-region

$$\Delta E_p = E_{Fp} - E_{Fi} = -k_B T \ln\left(\frac{N_a}{n_i}\right) \overset{\text{at room temp.}}{\approx} \underline{\underline{-0.480\ eV}}$$ below intrinsic level

b) Fermi level position is continous



From geometric consideration:

$$eV_0 + \cancel{E_g} = a + \Delta E_p + \Delta E_n + b \overset{a+b=E_g}{=} \cancel{E_g} + \Delta E_p + \Delta E_n$$

$$eV_0 = \Delta E_p + \Delta E_n \approx \underline{\underline{0.820\ eV}}$$

c) $eV_0 = k_B T \ln\left(\frac{N_a N_d}{n_i^2}\right) \approx \underline{\underline{0.820\ eV}}$ same as above

2) Shockley long diode equation

$$J = \left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a}\right) e n_i^2 \left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

Current:

$$I = JA = \left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a}\right) A e n_i^2 \left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

$$= I_0 \left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$$

Constant I_0: Minority carrier diffusion coefficients

$$D_p = \frac{k_B T}{e} \mu_p \approx 116{,}55 \ \mathrm{cm^2/s} \quad (n\text{-side})$$

$$D_n = \frac{k_B T}{e} \mu_n \approx 18{,}13 \ \mathrm{cm^2/s} \quad (p\text{-side})$$

Minority carrier diffusion lengths

$$L_p = \sqrt{D_p \tau_p} \approx 34{,}1 \cdot 10^{-3} \ \mathrm{cm}$$

$$L_n = \sqrt{D_n \tau_n} \approx 1{,}3 \cdot 10^{-3} \ \mathrm{cm}$$

$e = 1{,}6 \cdot 10^{-19}\ C$

$$\Rightarrow I_0 = \left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a}\right) A e n_i^2 \approx 5{,}68 \cdot 10^{-15}\ A$$

Unit: $C/s = A$

Diffusion current: at $V = 0{,}5V$:

$$I = I_0 \left[\exp\left(\frac{0{,}5\,eV}{0{,}0259\,eV}\right) - 1\right] \approx \underline{\underline{1{,}37 \cdot 10^{-6}\ A}}$$

at $V = -0{,}5V$

$$I = I_0 \left[\exp\left(\frac{-0{,}5\,eV}{0{,}0259\,eV}\right) - 1\right] \approx \underline{\underline{-5{,}68 \cdot 10^{-15}\ A}}$$

 Risto Montonen

Electron effective mass in GaAs

$m_e^* = 0{,}067 m_e \quad , m_e = 9{,}1 \cdot 10^{-31}\,kg$

Thermal velocity of electrons in CB of a nondegenerately doped GaAs at 300 K.

Electron wanders around the crystal with an average kinetic energy of $\frac{3}{2} k_B T$.

$$\Rightarrow \quad \frac{1}{\cancel{2}} m_e^* v_{th}^2 = \frac{3}{\cancel{2}} k_B T$$

$$v_{th} = \sqrt{\frac{3 k_B T}{m_e^*}} \quad , \quad k_B = 1{,}38 \cdot 10^{-23}\,J/K$$

$$\approx \underline{\underline{4{,}51 \cdot 10^5\,m/s}} \qquad \underline{\underline{\text{Fast}}}$$

Mean free time between electron scattering events:

$$\mu_e = \frac{e \tau_e}{m_e^*} \Leftrightarrow \tau_e = \frac{\mu_e m_e^*}{e} \quad , \quad \mu_e = 8500\,cm^2 V^{-1} s^{-1}, \quad e = 1{,}6 \cdot 10^{-19}\,C$$

$$\approx \underline{\underline{3{,}24 \cdot 10^{-13}\,s}}$$

$\underline{\underline{\text{Short time}}}$

$$\text{Unit: } \frac{m^2}{Vs} \cdot kg \cdot \frac{1}{C} = \frac{m^2}{Vs} \cdot kg \cdot \frac{1}{As} = \frac{Nm}{VA} = \frac{Nm}{W} = \frac{\cancel{Nm}}{\cancel{Nm}} s = s$$

Drift velocity of CB electrons in an applied E-field of $10^5\,V/m$:

$$v_d = \mu_e E = \underline{\underline{8{,}5 \cdot 10^4\,m/s}}$$

order of magnitude lower than v_{th}

4) Photon energy: $E = h\nu = h\frac{c}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$

Differential of λ with respect to E:

$$\frac{d\lambda}{dE} = -\frac{hc}{E^2} = -\frac{hc}{\left(\frac{hc}{\lambda}\right)^2} = -\frac{\lambda^2}{hc}$$

Sign does not matter for spreading

$\Rightarrow \left|\frac{d\lambda}{dE}\right| \approx \frac{\Delta\lambda}{\Delta E}$ linearity assumed

where $\Delta E \approx 3k_BT$.

$$\Rightarrow \frac{\Delta\lambda}{\Delta E} \approx \frac{\lambda^2}{hc}$$

$$\Delta\lambda \approx \frac{\lambda^2}{hc}\Delta E \approx \lambda^2 \frac{3k_BT}{hc} \quad \square$$

5) $E_g = 1{,}42$ eV, $T = 300$ K, $h = 4{,}135 \cdot 10^{-15}$ eVs

$$\lambda = \frac{hc}{E_g}$$

E_g depends on T

$$\Rightarrow \frac{d\lambda}{dT} = -\frac{hc}{E_g^2}\frac{dE_g}{dT}$$

Assuming linearity we have $\frac{d\lambda}{dT} \approx \frac{\Delta\lambda}{\Delta T}$

$$\Rightarrow \frac{\Delta\lambda}{\Delta T} \approx -\frac{hc}{E_g^2}\frac{dE_g}{dT}$$

$$\Delta\lambda \approx -\frac{hc}{E_g^2}\frac{dE_g}{dT}\Delta T$$

, $\frac{dE_g}{dT} \approx -4{,}5 \cdot 10^{-4}$ eV/K

$\Delta T = 10\,°C = 10$ K

$$\approx \underline{\underline{2{,}8 \text{ nm}}}$$

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) Width of the output spectrum is given by

$\Delta E = m k_B T$, where m is a numerical constant

a) Let's calculate $\Delta\lambda$ in the same way than in prob. 4.

$$\lambda = \frac{hc}{E}$$

sign does not matter for width

$$\frac{d\lambda}{dE} = -\frac{\lambda^2}{hc} \approx \frac{\Delta\lambda}{\Delta E} \Rightarrow \Delta\lambda = -\frac{\lambda^2}{hc}\Delta E = \underline{\underline{(-)\,\lambda^2 \frac{m k_B T}{hc}}}$$

b) Plot the values $\Delta\lambda$ as a function of λ.

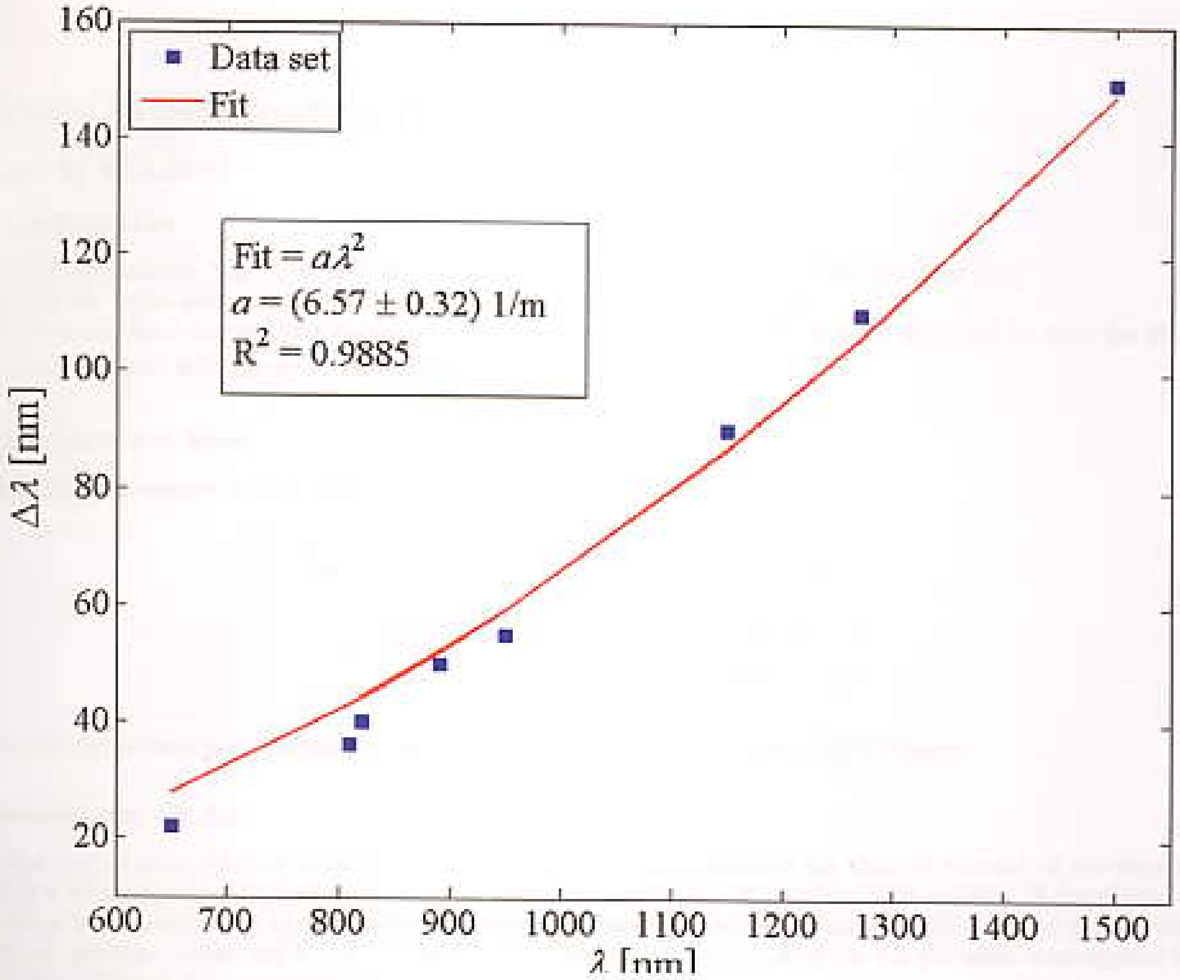
Fit a parabolic function $y = a\lambda^2$ to the data set. Fit results into a fit parameter

$$a = \frac{m k_B T}{hc} \approx (6{,}57 \pm 0{,}32)\cdot 10^4 \ 1/m \text{ , 95\% confidence level}$$

where $k_B T$ is assumed to be in room temp. and equals 0,0259 eV.

$h = 4{,}135 \cdot 10^{-15}$ eVs and $c = 3 \cdot 10^8$ m/s

$$\Rightarrow m = \frac{ahc}{k_B T} \approx \underline{\underline{3{,}15 \pm 0{,}16}} \text{ , at 95\% confidence level}$$



Data set
Fit
Fit = $a\lambda^2$
a = (6.57 ± 0.32) 1/m
R^2 = 0.9885
160
140
120
100
80
60
40
20
$\Delta\lambda$ [nm]
600
700
800
900
1000
1100
1200
1300
1400
1500
λ [nm]