1) Electrons in the CB of a nondegenerate semiconductor

a) Let's consider the energy distribution of electrons $n_E(E)$ in the conduction band (CB). Assuming that the density of state

$$g_{CB}(E) \propto (E-E_c)^{1/2}$$

and using Boltzmann statistics

$$f(E) \approx e^{-\frac{E-E_F}{k_B T}},$$

we show here that the energy distribution of the electrons in the CB can be written as

$$n_x(x) = C\sqrt{x}\, e^{-x},$$

where

$$x = \frac{E-E_c}{k_B T}$$

is the electron energy in terms of $k_B T$ measured from E_c and C is a constant at a given temperature (independent of E).

$$n_E(E) = g_{CB}(E) f(E) \qquad , \quad g_{CB}(E) \propto \sqrt{E-E_c}$$

$$\Rightarrow g_{CB}(E) = A\sqrt{E-E_c}$$

A: some constant

$$n_E(E) \approx A\sqrt{E-E_c}\, e^{-\frac{E-E_F}{k_B T}}$$

$$= A\sqrt{E-E_c}\, e^{-\frac{E}{k_B T}}\, e^{\frac{E_F}{k_B T}}$$

$$= A\sqrt{k_B T}\sqrt{\frac{E-E_c}{k_B T}}\, e^{-\frac{E}{k_B T}}\, e^{\frac{E_F}{k_B T}}\, e^{-\frac{E_c}{k_B T}}\, e^{\frac{E_c}{k_B T}}$$

$$= A\sqrt{k_B T}\, e^{-\frac{E_c-E_F}{k_B T}} \sqrt{\frac{E-E_c}{k_B T}}\, e^{-\frac{E-E_c}{k_B T}}$$

$$\Rightarrow \qquad n_x(x) = C\sqrt{x}\, e^{-x},$$

where $\quad C = A\sqrt{k_B T}\, e^{-\frac{E_F - E_c}{k_B T}} \quad$ and

$$x = \frac{E - E_c}{k_B T}. \qquad \square$$

b) Setting arbitrarily $C = 1$, see the attached figure.

Maximum of the curve:

$$\frac{dn_x(x)}{dx} = \frac{1}{2\sqrt{x}} e^{-x} - \sqrt{x}\, e^{-x} = 0$$

$$e^{-x}\left(\frac{1}{2\sqrt{x}} - \sqrt{x}\right) = 0 \quad \| : e^{-x} \neq 0$$

$$\frac{1}{2\sqrt{x}} - \sqrt{x} = 0$$

$$\frac{1}{2\sqrt{x}} = \sqrt{x} \quad \| \cdot \sqrt{x}$$

$$\underline{x = \frac{1}{2}}$$

$$n_{max} = n_{\frac{1}{2}}\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2}}\, e^{-\frac{1}{2}} \approx \underline{0{,}4289}$$

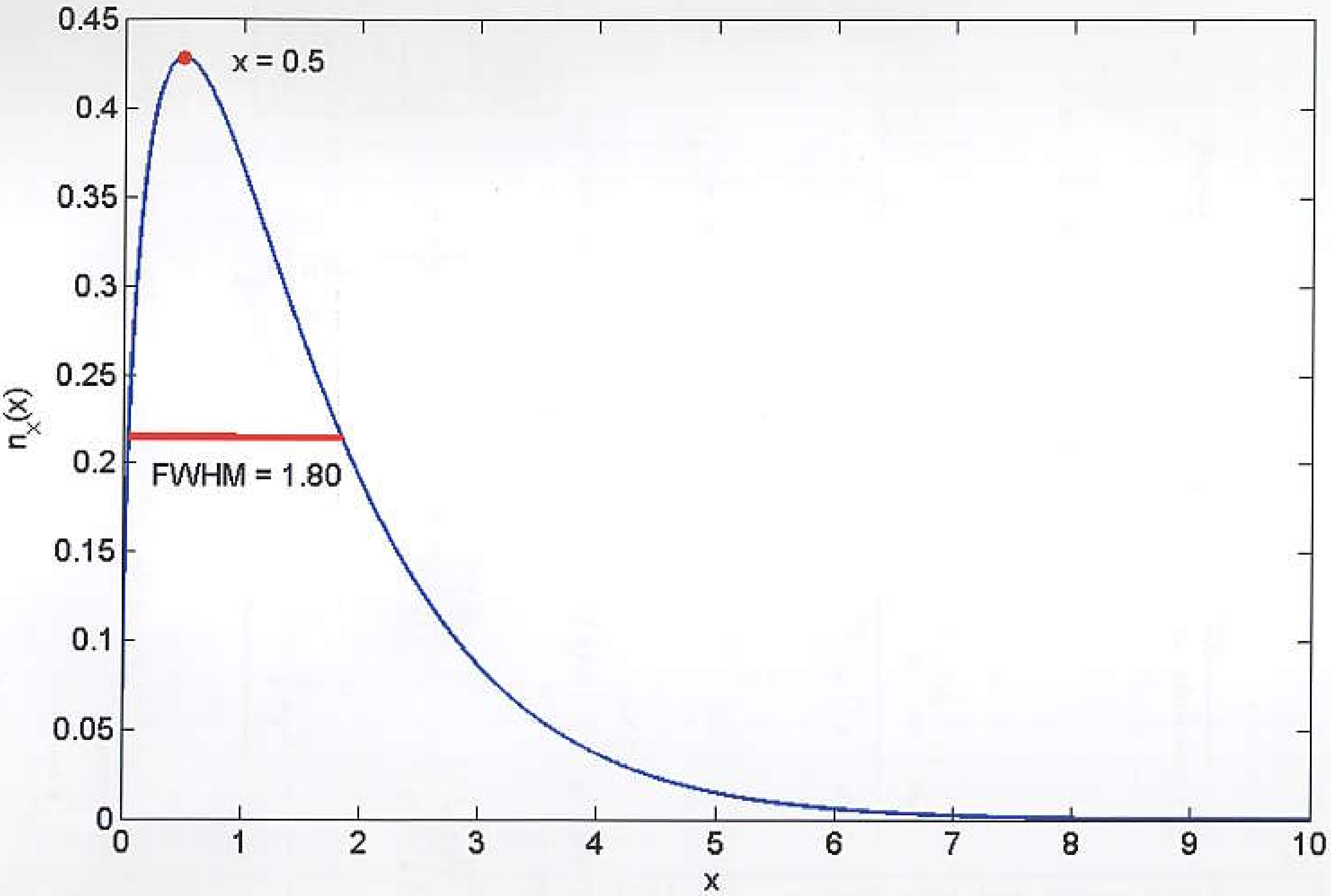
Full width at half maximum (FWHM) of the curve:

$$\sqrt{x}\, e^{-x} = \frac{n_{max}}{2}$$

Numerically: $x_1 \approx 0{,}051$ and $x_2 \approx 1.846$

$$\Rightarrow \text{FWHM} = x_2 - x_1 \approx 1{,}795 \approx \underline{1{,}8.}$$

$x = \frac{E - E_c}{k_B T} \Rightarrow$ energy from E_c in units of $k_B T$.



x = 0.5
FWHM = 1.80
$n_X(x)$
x

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c) Let's show that the average electron energy in the CB is $\frac{3}{2} k_B T$.

$$\text{Average electron energy} = \frac{\text{Total energy}}{\text{Number of electrons}}$$

$$\Rightarrow \hat{E} = \frac{\int_{E_c}^{E_c + \chi} E n_E(E)\, dE}{\int_{E_c}^{E_c+\chi} n_E(E)\, dE}$$

$$\Rightarrow \hat{x} = \frac{\int_0^{x_\chi} x\sqrt{x}\, e^{-x}\, dx}{\int_0^{x_\chi} \sqrt{x}\, e^{-x}\, dx} = \frac{\int_0^{x_\chi} x^{3/2} e^{-x}\, dx}{\int_0^{x_\chi} x^{1/2} e^{-x}\, dx}$$

NB! Average wavelength is calculated in a similar way from spectrum

$x_\chi \to \infty$

$$\Rightarrow \hat{x} = \frac{\int_0^{\infty} x^{3/2} e^{-x}\, dx}{\int_0^{\infty} x^{1/2} e^{-x}\, dx}, \quad \Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t}\, dt$$

$$= \frac{\int_0^{\infty} x^{5/2-1} e^{-x}\, dx}{\int_0^{\infty} x^{3/2-1} e^{-x}\, dx} = \frac{\Gamma(5/2)}{\Gamma(3/2)}, \quad \Gamma(1/2) = \sqrt{\pi}, \quad \Gamma(z+1) = z\Gamma(z)$$

$$\Gamma\left(\frac{5}{2}\right) = \Gamma\left(\frac{3}{2}+1\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{2}\Gamma\left(\frac{1}{2}+1\right) = \frac{3}{2}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)$$

$$= \frac{\frac{3}{2}\cdot\cancel{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}}{\cancel{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}} = \frac{3}{2} \Rightarrow \underline{\hat{E} = \frac{3}{2} k_B T}$$

d) $$x = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{E - E_c}{k_B T} \Rightarrow E = \underbrace{E_c + \frac{1}{2} k_B T}_{E_{max}}$$

Extrinsic n-Si

2) Si crystal has been doped n-type with $1 \cdot 10^{17}\ cm^{-3}$ phosphorus (P) donors. The electron drift mobility μ_e depends on the total concentration of ionized dopants N_{dopant}.

Conductivity of the crystal:

Dopant concentration $N_d = 10^{17}\ cm^{-3}$:
Drift mobility in Si for electrons $\mu_e = 730\ cm^2 V^{-1} s^{-1}$ and for holes $\mu_h = 328\ cm^2 V^{-1} s^{-1}$.

n-type conductivity:
Assuming $n = N_d$ and $p = \frac{n_i^2}{N_d}$ the conductivity becomes

$$\sigma = e N_d \mu_e + e \frac{n_i^2}{N_d} \mu_h \approx e N_d \mu_e$$

$e = 1{,}602 \cdot 10^{-19} C$
$n_i = 1{,}45 \cdot 10^{10}\ cm^{-3}$

$$\Rightarrow \quad \sigma \approx 11{,}69\ C cm^{-1} V^{-1} s^{-1} = \underline{\underline{11{,}69 \frac{1}{\Omega cm}}}$$

Fermi level with respect to the intrinsic crystal:

Intrinsic Fermi level:	$E_{Fi} = E_v + \frac{1}{2} E_g - \frac{1}{2} k_B T \ln \frac{N_c}{N_v}$	For notice, $E_{Fn} - E_{Fi}$ not calculated using this

$n_i = N_c e^{-\frac{E_c - E_{Fi}}{k_B T}}$, e concentration in CB for intrinsic Si

$N_d = n = N_c e^{-\frac{E_c - E_{Fn}}{k_B T}}$, e concentration in CB for doped Si

$$\Rightarrow \frac{N_d}{n_i} = \frac{\cancel{N_c} e^{-\frac{E_c - E_{Fn}}{k_B T}}}{\cancel{N_c} e^{-\frac{E_c - E_{Fi}}{k_B T}}} = e^{\frac{-\cancel{E_c} + E_{Fn} + \cancel{E_c} - E_{Fi}}{k_B T}} = e^{\frac{E_{Fn} - E_{Fi}}{k_B T}}$$

at room temp.

$$\Rightarrow \frac{E_{Fn} - E_{Fi}}{k_B T} = \ln \frac{N_d}{n_i} \quad \Rightarrow E_{Fn} - E_{Fi} = k_B T \ln \frac{N_d}{n_i} \approx \underline{\underline{0{,}4\,eV}}$$

above intrinsic Fermi level

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3) Compensation doping in n-type Si

N-type Si sample has been doped with 10^{16} phosphorus (P) atoms per cm^3

a) Electron and hole concentration: n-type

Electron conc. : $n \approx N_d = 10^{16}\ cm^{-3}$

Hole conc. : $p = \frac{n_i^2}{N_d} = \frac{(1{,}45 \cdot 10^{10}\ cm^{-3})^2}{10^{16}\ cm^{-3}} \approx 21000\ cm^{-3}$

b) Room temperature conductivity of the sample:

$$\sigma = eN_d\mu_e + e\frac{n_i^2}{N_d}\mu_h$$

$e = 1{,}602 \cdot 10^{-19}\ C$, $\mu_e = 1200\ cm^2V^{-1}s^{-1}$, $\mu_h = 444\ cm^2V^{-1}s^{-1}$

$\Rightarrow \sigma = 1{,}92\ \Omega^{-1}cm^{-1}$

c) Fermi level with respect to E_{Fi} :

$$n_i = N_c e^{-\frac{E_c - E_{Fi}}{k_BT}}$$

$$N_d = n = N_c e^{-\frac{E_c - E_{Fn}}{k_BT}}$$

$$\frac{N_d}{n_i} = \frac{\cancel{N_c} e^{-\frac{E_c - E_{Fn}}{k_BT}}}{\cancel{N_c} e^{-\frac{E_c - E_{Fi}}{k_BT}}} = e^{\frac{E_{Fn} - E_{Fi}}{k_BT}}$$

at room temp.

$\Rightarrow E_{Fn} - E_{Fi} = k_BT \ln\frac{N_d}{n_i} \approx 0{,}35\ eV$ above intrinsic Fermi level

d) The crystal is now doped with 10^{17} boron acceptors per cm^3. Electron and hole concentrations:

$$N_a = 10^{17}\,cm^{-3} > N_d$$

$\Rightarrow$ Hole conc.: $p = N_a - N_d = \underline{\underline{9 \cdot 10^{16}\,cm^{-3}}}$

Electron conc.: $n = \dfrac{n_i^2}{p} \approx \underline{\underline{2340\,cm^{-3}}}$

e) Fermi level with respect to E_{Fi}:

Intrinsic: , Doped:

$$p_i = n_i \approx N_v e^{-\frac{E_{Fi}-E_v}{k_B T}} , \quad p = N_a = N_v e^{-\frac{E_{Fp}-E_v}{k_B T}}$$

$$\Rightarrow \frac{p}{n_i} = \frac{\cancel{N_v} e^{-\frac{E_{Fp}-E_v}{k_B T}}}{\cancel{N_v} e^{-\frac{E_{Fi}-E_v}{k_B T}}} = e^{-\frac{E_{Fp}-E_{Fi}}{k_B T}}$$

at room temp.

$$\Rightarrow E_{Fp} - E_{Fi} = -k_B T \ln \frac{p}{n_i} \approx \underline{\underline{-0{,}81\,eV}}$$ below intrinsic Fermi level

4) Si pn junction

Let's consider a long pn junction diode with an acceptor doping, N_a, of $10^{18}\,cm^{-3}$ on the p-side and donor concentration, N_d, on the n-side. The diode is forward biased with a voltage of 0,6 V across it. The diode cross-sectional area is $1\,mm^2$. The minority carrier recombination time, τ, depends on the dopant concentration, N_{dopant} (cm^{-3}), through the following approximate empirical relation $\tau \approx (5 \cdot 10^{-7}) / (1 + 2 \cdot 10^{-17} N_{dopant})$ in seconds.

a) Let's suppose that $N_d = 10^{15}\,cm^{-3}$. The depletion layer extends essentially into the n-side and we need to consider minority carrier recombination time, τ_h, in this region.

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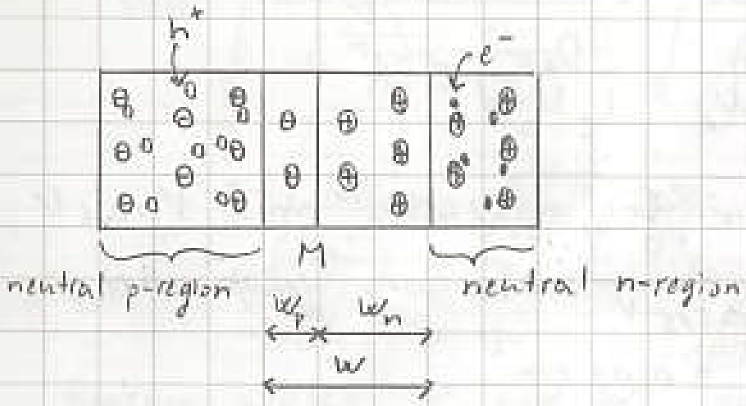
Let's calculate the diffusion and recombination contributions to the total diode current with $N_a = 10^{18}\ cm^{-3}$.

Since $N_a >> N_d$ this diode is p^+n diode and $W_n > W_p$



Hole lifetime τ_h in the n-side: $N_{dopant} = 10^{15}$

$$\tau_h = \frac{5 \cdot 10^{-7}}{1+2 \cdot 10^{-17} \cdot 10^{15}}\ s \approx 490,2\ ns$$

Electron lifetime τ_e in the p-side: $N_{dopant} = 10^{18}$

$$\tau_e = \frac{5 \cdot 10^{-7}}{1+2 \cdot 10^{-17} \cdot 10^{18}}\ s \approx 23,8\ ns$$

Diffusion component of the diode current:

Diffusion coefficients of holes and electrons:
$\mu_h = 478\ cm^2 V^{-1} s^{-1}$, $\mu_e = 280\ cm^2 V^{-1} s^{-1}$

$$D_h = \frac{k_B T \mu_h}{e} \approx 12,356\ cm^2 s^{-1}$$

$$D_e = \frac{k_B T \mu_e}{e} \approx 7,238\ cm^2 s^{-1}$$

Diffusion lengths:

into n-side: $L_h = \sqrt{D_h \tau_h} \approx 2,46 \cdot 10^{-3}\ cm = 24,6\ \mu m$

into p-side: $L_e = \sqrt{D_e \tau_e} \approx 0,41 \cdot 10^{-3}\ cm = 4,1\ \mu m$

Thus diffusion component of the diode current is:

$$I_D = A J_0 = Aen_i^2\left(\frac{D_h}{L_h N_d} + \frac{D_e}{L_e N_a}\right)\underbrace{\left(e^{\frac{eV}{k_B T}} - 1\right)}_{\approx e^{\frac{eV}{k_B T}}}$$

$$\approx Aen_i^2\left(\frac{D_h}{L_h N_d} + \frac{D_e}{L_e N_a}\right)e^{\frac{eV}{k_B T}}$$

$e = 1{,}602 \cdot 10^{-19}\,C$, $n_i = 1{,}45\ 10^{10}\,cm^{-3}$, $V = 0{,}6V$,

$\frac{e}{k_B T} = \frac{1}{0{,}02585\,V}$ at room temperature,

$A = 1mm^2 = 0{,}01\,cm^2$

$$\Rightarrow I \approx 0{,}02042\,A = \underline{20{,}42mA}$$

$$\text{Unit: } cm^2 \cdot C(cm^{-3})^2 \cdot \frac{\frac{cm^2}{s}}{cm \cdot cm^{-3}}$$

$$= cm^2 \cdot C \cdot cm^{-6} \cdot \frac{cm^2}{cm^{-2}s}$$

$$= \cancel{cm^2} \cdot C \cdot \cancel{cm^{-6}} \cdot \frac{\cancel{cm^4}}{s} = \frac{C}{s} = A$$

Recombination component of the diode current:

Built in potential:

$$V_0 = \frac{k_B T}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) \overset{\text{at room temp.}}{\approx} 0{,}75V$$

Knowing built in potential we are able to calculate the total width of the depletion region.

$$V_0 - V = \frac{eN_a N_d W_0^2}{2\varepsilon(N_a + N_d)} \Rightarrow W_0 = \sqrt{\frac{2\varepsilon(N_a + N_d)(V_0 - V)}{eN_a N_d}},$$

where $\varepsilon = \varepsilon_0\varepsilon_r = 1{,}034 \cdot 10^{-10}\,Fm^{-1} = 1{,}034 \cdot 10^{-12}\,F\,cm^{-1}$ for silicon Si.

$$\Rightarrow W_0 \approx 4{,}47\cdot 10^{-5}\ cm = 0{,}447\ \mu m$$

$$\text{Unit: } \sqrt{\frac{\frac{F}{cm}\cdot\frac{1}{cm^3}\cdot V}{C\cdot\frac{1}{cm^3}\cdot\frac{1}{cm^3}}} = \sqrt{\frac{\frac{\cancel{C}}{\cancel{V}cm}\cdot\frac{1}{cm^3}\cdot\cancel{V}}{\cancel{C}\cdot\frac{1}{cm^6}}}$$

$$= \sqrt{\frac{\frac{1}{cm^4}}{\frac{1}{cm^6}}} = \sqrt{cm^2} = cm$$

Depletion widths, W_p and W_n, can be calculated with N_a and N_d:

$$N_a W_p = N_d W_n \quad , \quad W_p + W_n = W_0 \Rightarrow W_n = W_0 - W_p$$

$$N_a W_p = N_d (W_0 - W_p)$$

$$W_p (N_a + N_d) = N_d W_0$$

$$W_p = \frac{N_d W_0}{N_a + N_d} \approx 4{,}46\cdot 10^{-8}\ cm = 0{,}446\ nm$$

$$W_n = W_0 - W_p \approx 4{,}47\cdot 10^{-5}\ cm = 0{,}447\ \mu m$$

Thus the recombination component of the diode current is:

$$I_R = A J_R = A\frac{e n_i}{2}\left[\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h}\right]\underbrace{\left[e^{\frac{eV}{2k_BT}} - 1\right]}_{\approx e^{\frac{eV}{2k_BT}}}$$

$$\approx \frac{A e n_i}{2}\left[\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h}\right] e^{\frac{eV}{2k_BT}} \approx 0{,}00012\ A = \underline{0{,}12\ mA}$$

$$\text{Unit: } \cancel{cm^2}\cdot C \cdot \frac{1}{\cancel{cm^3}}\cdot\frac{\cancel{cm}}{s} = \frac{C}{s} = A$$

Conclusion: Diode current is dominated by the diffusion component, $I_D \gg I_R$.

b) Let's suppose that $N_d = N_A = 10^{18}\ cm^{-3}$. Then W extends equally to both sides and further, $\tau_e = \tau_h$.

Let's calculate the diffusion and recombination contributions for this diode with the same procedure that in the previos case.

Hole and electron life time: $\tau_e = \tau_h \approx 23{,}8\ ns$

Hole and electron diffusion coefficients: $D_h \approx 12{,}356\ cm^2 s^{-1}$
$D_e \approx 7{,}238\ cm^2 s^{-1}$

Diffusion lengths: $L_h = \sqrt{D_h \tau_h} \approx 0{,}54 \cdot 10^{-3}\ cm = 5{,}4\ \mu m$
$L_e = \sqrt{D_e \tau_e} \approx 0{,}41 \cdot 10^{-3}\ cm = 4{,}1\ \mu m$

Diffusion component of the diode current:

$$I_D = A J_D = A e n_i^2 \left(\frac{D_h}{L_h N_d} + \frac{D_e}{L_e N_A} \right) e^{\frac{eV}{k_B T}} \approx 1{,}62 \cdot 10^{-4}\ A = \underline{0{,}162\ mA}$$

Built in potential: $V_0 = \frac{k_B T}{e} \ln\left(\frac{N_A N_d}{n_i^2} \right) \approx 0{,}93\ V$ at room temp

Total width of the depletion region:

$$W_0 = \sqrt{\frac{2 \varepsilon (N_a + N_d)(V_0 - V)}{e N_a N_d}} \approx 2{,}93 \cdot 10^{-6}\ cm = 29{,}3\ nm$$

Depletion widths, W_p and W_n: $W_p = W_n \approx 1{,}47 \cdot 10^{-6}\ cm$
$= 14{,}7\ nm$

Recombination component of the diode current:

$$I_R = A J_R = \frac{A e n_i}{2} \left[\frac{W_p}{\tau_e} + \frac{W_e}{\tau_h} \right] e^{\frac{eV}{2 k_B T}} \approx 1{,}57 \cdot 10^{-4}\ A = \underline{0{,}157\ mA}$$

Conclusion: Diffusion and recombination components are nearly equal.