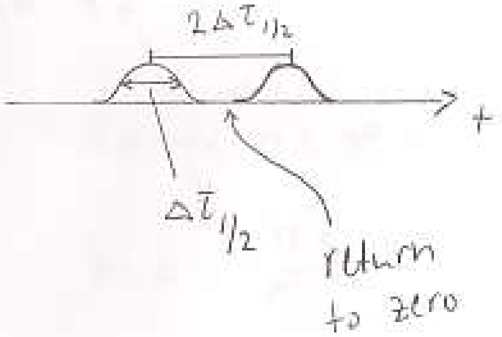
 Risto Montonen

1 a) If a short light pulse spreads temporally to a FWHM-width of $\Delta\tau_{1/2}$ due to dispersion the max bitrate can be estimated as

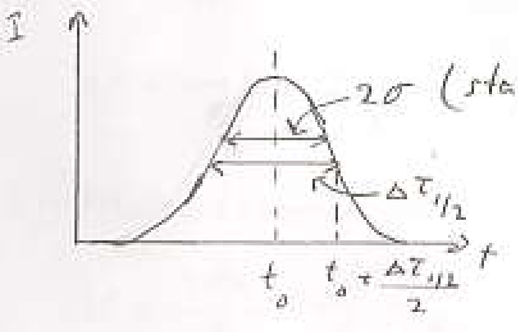


$$B \approx \frac{1}{2\Delta\tau_{1/2}} = \frac{0{,}5}{\Delta\tau_{1/2}}$$

In ex. 4 prob. 3 we got a result $\Delta\tau_{1/2} \approx 1{,}69$ ns

$$\Rightarrow B \approx 2{,}96 \cdot 10^8 \ 1/s = 296 \ \text{Mbit/s}$$

b) Assuming Gaussian pulse shape:



$$I = I_0 \exp\left[-\frac{(t-t_0)^2}{2\sigma^2}\right]$$

At $t = t_0 + \frac{\Delta\tau_{1/2}}{2}$ $\quad I = \frac{I_0}{2}$

$$\Rightarrow \frac{I_0}{2} = I_0 \exp\left[-\frac{\left(t_0 + \frac{\Delta\tau_{1/2}}{2} - t_0\right)^2}{2\sigma^2}\right]$$

$$\frac{1}{2} = \exp\left[-\frac{\Delta\tau_{1/2}^2}{8\sigma^2}\right] \qquad \| \ln(\,)$$

$$-\ln 2 = -\frac{\Delta\tau_{1/2}^2}{8\sigma^2}$$

$$\Rightarrow \quad \sigma = \pm \sqrt{\frac{\Delta\tau_{1/2}^2}{8\ln 2}} = \frac{\Delta\tau_{1/2}}{2\sqrt{2\ln 2}} \approx 0{,}425\,\Delta\tau_{1/2}$$

$$\Rightarrow \quad B \approx \frac{0.5}{\Delta\tau_{1/2}} \approx \frac{0{,}5}{\frac{\sigma}{0{,}425}} \approx \frac{0{,}21}{\sigma} \approx \underline{\underline{\frac{0{,}25}{\sigma}}} \quad \square$$

(↑ Pulses 4σ apart)

2) $\lambda = 850\ nm$, $n_f = 1{,}475$, $n_c = 1{,}455$, $2a = 100\ \mu m$

a) $$V = \frac{2\pi a}{\lambda}\left(n_f^2 - n_c^2\right)^{1/2} \approx \underline{\underline{89{,}47}}$$

$$M \approx \frac{V^2}{2} \approx \underline{\underline{4002}}$$

b) Number of modes drop down to one when

$$V = \frac{2\pi a}{\lambda}\left(n_f^2 - n_c^2\right)^{1/2} < 2{,}405$$

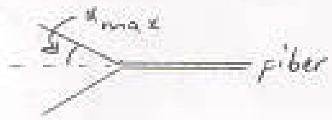
$$\Rightarrow \lambda > \frac{2\pi a}{2{,}405}\left(n_f^2 - n_c^2\right)^{1/2} \approx \underline{\underline{31{,}6\ \mu m}}$$

c) $$NA = \left(n_f^2 - n_c^2\right)^{1/2} \approx \underline{\underline{0{,}24}}$$

d) Maximum acceptance angle

$$\alpha_{max} = \sin^{-1}\frac{NA}{n_0} = \sin^{-1} NA \approx \underline{\underline{14^\circ}} \quad \text{(Half angle)}$$

$n_0 = n_{air} \approx 1$



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e) Modal dispersion between extreme modes:

$$\frac{\Delta\tau_{intermodal}}{L} \approx \frac{n_f - n_c}{c} \approx 6,67 \cdot 10^{-11}\ s/m \qquad c \approx 3,0 \cdot 10^8\ m/s$$
$$= 66,7\ ns/km$$

Material dispersion (chromatic or intramodal) can be included

$$\Delta\tau^2 = \Delta\tau_{intermodal}^2 + \Delta\tau_{intramodal}^2$$

Bit rate x distance product. Gaussian pulse

$$BL \approx \frac{0,25\,L}{\sigma_{intermodal}} \quad , \sigma \approx 0,425\,\Delta\tau$$

$$\approx \frac{0,25\,L}{0,425\,\Delta\tau_{intermodal}} \approx 8,8\ Mbit/s \cdot km$$

3) $\lambda = 1300\ nm$, $\Delta\lambda = 2\ nm$, $n_f = 1,468$, $n_c = 1,464$, $2a = 8\ \mu m$

a) $V = \frac{2\pi a}{\lambda}(n_f^2 - n_c^2)^{1/2} \approx 2,094 < 2,405$

$\Rightarrow$ single mode fiber

b) Fiber becomes multimode when

$$V = \frac{2\pi a}{\lambda}(n_f^2 - n_c^2)^{1/2} > 2,405$$

$$\Rightarrow \lambda < \frac{2\pi a}{2,405}(n_f^2 - n_c^2)^{1/2} \approx 1130\ nm$$

c) $NA = (n_f^2 - n_c^2)^{1/2} \approx 0,11$

d) $\alpha_{max} = \sin^{-1} NA \approx 6,2°$, in air

c) Material dispersion coefficient, $D_m \approx -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$

$D_m = -7{,}5 \text{ ps km}^{-1}\text{nm}^{-1}$

Waveguide dispersion coefficient, $D_w \approx -\frac{0.025\,\lambda}{a^2 c n_c}$, $1{,}5 < V < 2{,}4$

$D_w = -5 \text{ ps km}^{-1}\text{nm}^{-1}$

Chromatic dispersion

$$\frac{\Delta\tau_{ch}}{L} = |D_m + D_w|\,\Delta\lambda = 25 \text{ ps/km}$$

Bit rate × distance product: Gaussian pulse

$$BL \approx \frac{0{,}25}{0{,}425}\frac{L}{\Delta\tau_{ch}} \approx 23{,}5 \text{ Gbit/s} \cdot \text{km}$$

4) Graded index (GRIN) fiber has an index profile

$$n = n_f[1 - 2\Delta(r/a)^\gamma], \quad r < a \qquad n_f = 1{,}480$$

$$n = n_c, \quad r \geq a \qquad n_c = 1{,}460$$

$$\gamma = 2(1-\Delta) \quad \text{and} \quad \Delta = \frac{n_f - n_c}{n_f}$$

The intermodal dispersion is given by

$$\frac{\sigma}{L} \approx \frac{n_f}{20\sqrt{3}\,c}\Delta^2 \approx 2{,}60 \cdot 10^{-14} \text{ s/m} = 0{,}026 \text{ ns/km}$$

Bit rate × distance product:

$$BL \approx \frac{0{,}25L}{\sigma} \approx 9{,}6 \text{ Gbit/s} \cdot \text{km}$$

This is about 1000 times greater than in MM fiber and about half than in SM fiber.