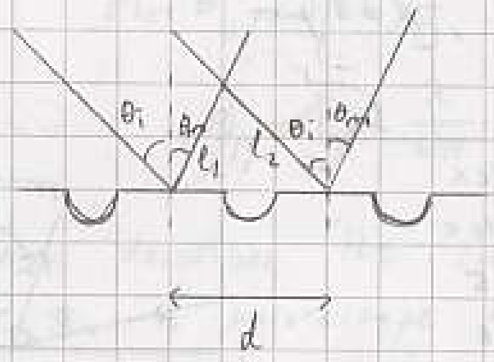
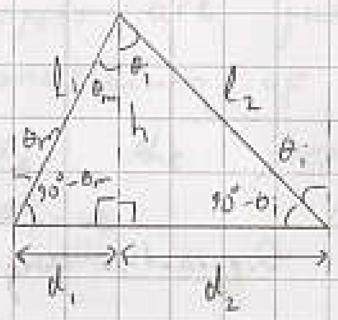
Photonics 17 Risto Montonen

1) Reflection grating, groove separation $= d$





$d_1 + d_2 = d$

$$\cos(90° - \theta_m) = \frac{d_1}{l_1} = \sin\theta_m \quad , \quad \cos(90° - \theta_i) = \frac{d_2}{l_2} = \sin\theta_i$$

Assuming $\theta_m + \theta_i \approx 90°$

$$\Rightarrow \cos(90° - \theta_m) \approx \frac{l_1}{d} \quad , \quad \cos(90° - \theta_i) \approx \frac{l_2}{d}$$

$$\Rightarrow l_1 \approx d\cos(90° - \theta_m) \quad , \quad l_2 \approx d\cos(90° - \theta_i)$$

$$\Rightarrow l_1 \approx d\sin\theta_m \quad , \quad l_2 \approx d\sin\theta_i$$

Condition for constructive interference:

$$\Delta l = l_1 - l_2 \approx d(\sin\theta_m - \sin\theta_i) = m\lambda \quad , \quad m = 0, \pm1, \pm2, \ldots$$

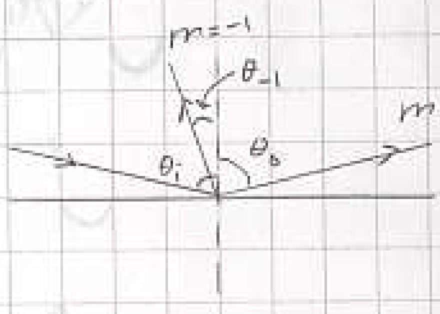
Diffraction angle: $\sin\theta_m = \frac{m\lambda}{d} + \sin\theta_i$

$$\theta_m = \sin^{-1}\left[\frac{m\lambda}{d} + \sin\theta_i\right]$$

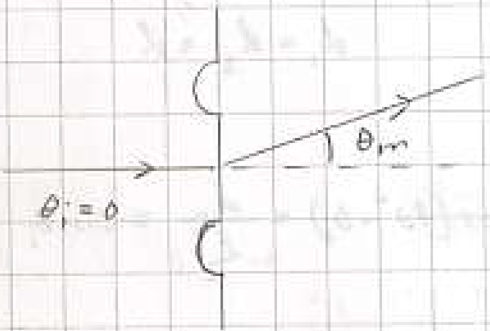
$\theta_i = 89^0$, $\lambda = 1{,}3\,\mu m$, $d = 1\,\mu m$

=>

m	θ_m
-3	complex
-2	complex
-1	-17,5
0	89,0
1	complex
2	complex
3	complex



2)



Diffraction angle:

$$\theta_m = \sin^{-1}\left[\frac{m\lambda}{d} + \sin\theta_i\right], \theta_i = 0$$
$$= \sin^{-1}\left[\frac{m\lambda}{d}\right]$$

Angular separation: $\Delta\theta_m = \theta_m(\lambda_1) - \theta_m(\lambda_2)$

$d = 2\,\mu m$, $\lambda_1 = 1{,}55\,\mu m$, $\lambda_2 = 1{,}54\,\mu m$

Gives real valued results for $m=0$ and $m=\pm 1$ modes. Let's consider $m=1$ then $\Delta\theta_1 \approx \underline{\underline{0{,}45^\circ}}$.

Considering the argument $m\lambda/d$ greater wavelength separation is achieved by decreasing d. However, $m\lambda$ sets a limit for d.

3) a) Airy disk diameter is defined as the diameter of the first dark ring in Airy diffraction pattern.

$$\sin\theta_0 = 1{,}22\,\frac{\lambda}{D}$$, considering objective $\theta_0 = \theta_{max} = \sin^{-1} NA$

$$D = 1{,}22\,\frac{\lambda}{NA}$$

Rayleigh criterion describes the smallest resolvable lateral feature in a microscope image. Rayleigh criterion is defined by the Airy disk radius so that two spots are just resolvable when the principal maximum of one diffraction pattern coincides the first dark ring of the other. Thus we have

$$d = 0{,}61\,\frac{\lambda}{NA} ,$$

where d is the smallest resolvable lateral feature.

b) 50x objective has typically a numerical aperture $NA = 0{,}55$.

$$\Rightarrow\ d(\lambda = 250\,\text{nm}) \approx 280\,\text{nm}$$
$$d(\lambda = 350\,\text{nm}) \approx 390\,\text{nm}$$
$$d(\lambda = 750\,\text{nm}) \approx 830\,\text{nm}$$

Blue-ray disk has a groove separation approximately 320 nm. Applying $NA = 0{,}55$ objective only UV illumination would see the grooves. However UV microscopy is not feasible due to strong light absorption at short wavelengths in typical optical components.

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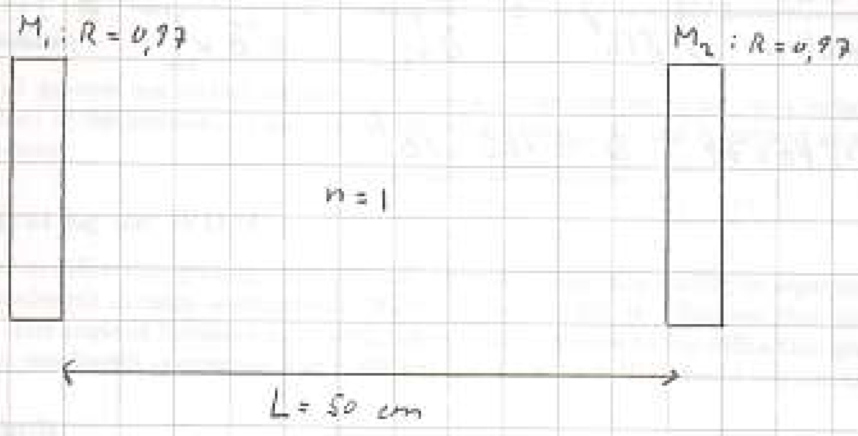
4) Fabry-Perot optical cavity



Let's consider wavelength $\lambda = 632,8$ nm.

Nearest mode number:

$$\underbrace{m\left(\frac{\lambda}{2}\right) = L}_{\text{standing wave condition}} \Rightarrow m = \frac{2L}{\lambda} = 1\,580\,278,12\ldots \approx \underline{\underline{1\,580\,278}}$$

Wavelength of this mode:

$$\lambda_m = \frac{2L}{m} \approx 632,8000516\ldots \text{ nm} \approx \underline{\underline{632\,800\,052 \text{ fm}}}$$

Mode separation in frequency: $c = 2,9979 \cdot 10^8$ m/s

$$\Delta\nu_m = \frac{c}{2L} = 299,79 \text{ MHz}$$

Mode separation in wavelength:

$$\Delta\lambda_m = \frac{^{m+1)}2L}{m} - \frac{^{m)}2L}{m+1} = \frac{2L(\cancel{m}+1-\cancel{m})}{m(m+1)} = \underline{\underline{\frac{2L}{m(m+1)}}}$$

Finesse: $F = \frac{\pi R^{1/2}}{1-R} \approx \underline{103}$

Also $F = \frac{\Delta\nu_m}{\delta\nu_m}$

mode separation

spectral (frequency) width of mode

Q-factor:

$$Q = \frac{\text{Resonant frequency}}{\text{Spectral width}} = \frac{\nu_m}{\delta\nu_m} = \frac{m\Delta\nu_m}{\delta\nu_m} = \underline{\underline{mF}}$$

At $m = 1580278$ $\underline{\underline{Q \approx 163 \cdot 10^{6}}}$

Mode separation in frequency:

$$\Delta\nu_m = \nu_{m+1} - \nu_m = \frac{c}{\lambda_{m+1}} - \frac{c}{\lambda_m} = \frac{c}{\frac{2L}{m+1}} - \frac{c}{\frac{2L}{m}}$$

$$= \frac{c}{2L}(m+1-m) = \frac{c}{2L}$$

Mode separation in wavelength:

$$\Delta\lambda = \Delta\nu \frac{\lambda_i^2}{c} = \frac{\cancel{c}}{2L}\frac{\lambda_i^2}{\cancel{c}} = \frac{\lambda_i^2}{2L}$$

at 632,8nm $\Delta\lambda \approx 0{,}4$ pm