 Risto Montonen

1) $n_f = 1{,}46$ $\Delta = 0{,}003$ $d_f = 8\,\mu m$

a) $NA = \sqrt{n_f^2 - n_c^2}$

$$\Delta = \frac{(n_f+n_c)\,n_f - n_c}{n_f} = \frac{n_f^2 - n_c^2}{n_f(n_f+n_c)} = \frac{n_f^2 - n_c^2}{n_f^2 + n_f n_c}, \quad n_f \approx n_c$$

$$\approx \frac{n_f^2 - n_c^2}{2n_f^2} \Rightarrow n_f^2 - n_c^2 = 2n_f^2\Delta$$

$$\Rightarrow NA = \sqrt{2n_f^2\Delta} \approx \underline{0{,}113}$$

b) $\sin\alpha_{max} = \frac{NA}{n_0}$, $n_0 = n_{air} \approx 1$

$$\alpha_{max} = \sin^{-1}(NA) \approx \underline{6{,}5°}$$

c) Conditions for single mode propagation:

- Cladding needs to be thick enough to prevent mode leaking. 10 times core diameter is usually sufficient. Because of evanescent wave.

  $10 \cdot 8\,\mu m = 80\,\mu m < d_c = 125\,\mu m \Rightarrow$ OK

- Single mode propagation is achieved when $V \leq 2{,}405$, where V is a normalized frequency or a normalized thickness of the fiber core.

In these conditions only LP_{10} mode will propagate.

$$V = \frac{2\pi a}{\lambda_c}\sqrt{n_f^2 - n_c^2} \leq 2{,}405, \quad a = \frac{d_f}{2}$$

$$\lambda_c \geq \frac{2\pi a}{2{,}405} NA \approx \underline{1{,}18\,\mu m}$$

(≥: to maintain single mode propagation)

2) Group velocity describes the rate of propagation of a wave packet, and is defined as

$$v_g = \frac{d\omega}{dk}$$

In a medium we have

$$k = \frac{2\pi n}{\lambda} \quad \text{and} \quad \omega = kv = \frac{2\pi \not{n}}{\lambda}\frac{c}{\not{n}} = \frac{2\pi c}{\lambda},$$

where λ is in vacuum.

Using chain rule for the group velocity we get

$$v_g = \frac{d\omega}{dk} = \frac{d\omega}{d\lambda}\frac{d\lambda}{dk} = \frac{d\omega}{d\lambda}\left(\frac{dk}{d\lambda}\right)^{-1}, \quad n \text{ depends on wavelength}$$

$$= -\frac{2\pi c}{\lambda^2}\left(-\frac{2\pi n}{\lambda^2} + \frac{2\pi}{\lambda}\frac{dn}{d\lambda}\right)^{-1} = -\frac{2\pi c}{\lambda^2}\frac{1}{2\pi\left(-\frac{n}{\lambda^2} + \frac{1}{\lambda}\frac{dn}{d\lambda}\right)}$$

From Sellmeier equation

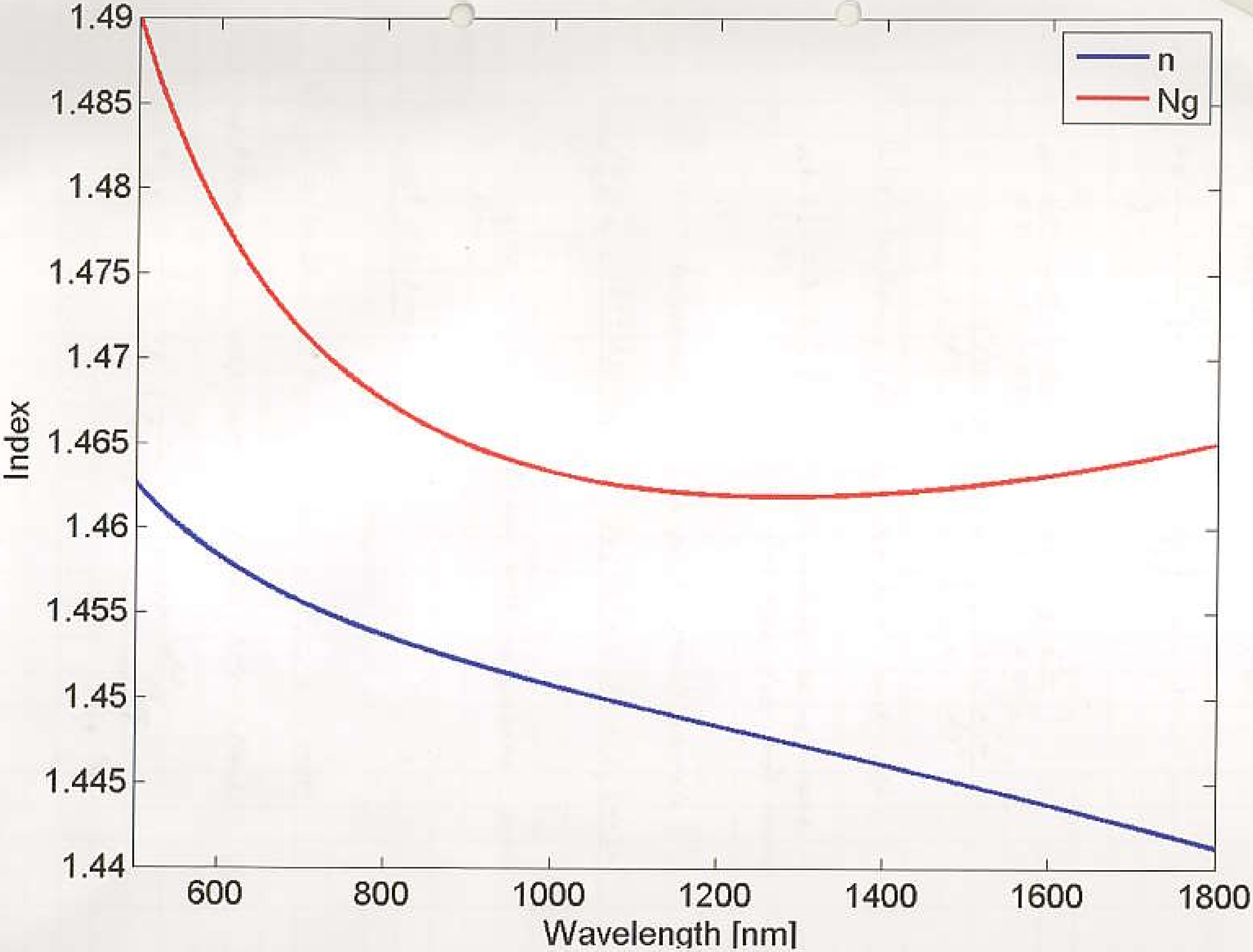
$$= \frac{c}{n - \lambda\frac{dn}{d\lambda}} = \frac{c}{N_g} \Rightarrow N_g = n - \lambda\frac{dn}{d\lambda}$$

Derivative $dn/d\lambda$ can be calculated for example by a five-point stencil numerical derivative

$$f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h},$$

where h is the minimum wavelength step.

See the result from the attached figure.



n
Ng
Index
1.49
1.485
1.48
1.475
1.47
1.465
1.46
1.455
1.45
1.445
1.44
600
800
1000
1200
1400
1600
1800
Wavelength [nm]

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3) Group propagation in a pure silica fiber:

$$t = \frac{L}{v_g} = \frac{L N_g}{c} = \frac{L}{c}\left(n - \lambda \frac{dn}{d\lambda}\right)$$

$$\Rightarrow \frac{dt}{d\lambda} = \frac{L}{c}\left[\frac{dn}{d\lambda} - \left(\frac{d\lambda}{d\lambda}\frac{dn}{d\lambda} + \lambda \frac{d^2 n}{d\lambda^2}\right)\right]$$

$$= \frac{L}{c}\left[\frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda \frac{d^2 n}{d\lambda^2}\right] = -\frac{L}{c}\lambda \frac{d^2 n}{d\lambda^2}$$

Pulse lengthening due to non-zero bandwidth:

$$\Delta t = \left| -\frac{L}{c}\lambda \frac{d^2 n}{d\lambda^2}\right| \Delta\lambda$$, absolute value since direction does not matter

Numerical derivation by five-point stencil formula:

$$f''(x) \approx \frac{-f(x+2h) + 16 f(x+h) - 30 f(x) + 16 f(x-h) - f(x-2h)}{12 h^2}.$$

This gives us at $\lambda = 850$ nm with $\Delta\lambda = 20$ nm and $L = 1$ km

$\Delta t \approx 1{,}69$ ns

4) $L = 130$ km $P_{in} = 1$ mW $P_{out} = 10$ nW

a) Attenuation coefficient (power attenuation):

$$\alpha_{dB} = \frac{1}{L} 10 \log_{10}\left(\frac{P_{in}}{P_{out}}\right) \approx 0{,}385 \ dB/km$$

b) Attenuation in a glass fiber due to Rayleigh scattering is approximately

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4}(n^2-1)\beta_T k_B T_f$$

$\lambda = 1{,}55\ \mu m$ wavelength
$n(\lambda = 1{,}55\ \mu m) = 1{,}444$ refractive index
$k_B = 1{,}3807 \cdot 10^{-23}\ J/K$ Boltzmann constant
T_f = fictive temperature (roughly the softening temperature)
$= 1730\ °C \approx 2003\ K$
$\beta_T(T_f = 1730\ °C) = 7 \cdot 10^{-11}\ m^2/N$ isothermal compressibility

$$\Rightarrow \alpha_R \approx 3{,}01 \cdot 10^{-5}\ 1/m = 3{,}01 \cdot 10^{-2}\ 1/km$$

Unit: $[\alpha_R] = \frac{1}{m^4}\frac{m^2}{N}\frac{J}{K}K = \frac{1}{m^2}\frac{J}{N} = \frac{1}{m^2}\frac{Nm}{N} = \frac{1}{m}$

Conversion from $1/m$ to dB/m

$$\alpha_{dB} = \frac{10}{\ln(10)}\alpha_R \approx 0{,}131\ dB/km$$

Rayleigh scattering attenuates the light signal due to small inhomogeneous regions in which the refractive index is different than the one in the surrounding medium. These regions act like small dielectric particles which scatter the propagating wave in different directions. In this type of scattering, the inhomogeneous region size is typically smaller than one-tenth of the wavelength.