1) a) \[ c = \lambda_0 v \Rightarrow \lambda_0 = \frac{c}{v} \Rightarrow v = \frac{c}{\lambda_0} \] 

Optical frequency

Differentiation:

\[ \Delta \lambda_0 = -\frac{c}{v} \Delta v = -\frac{c}{v} \Delta v \Rightarrow \Delta \lambda_0 = \frac{\lambda_0^2}{c} \Delta v \]

Temporal coherence time

\[ \Delta t = \frac{1}{\Delta v} \Rightarrow \Delta \lambda_0 = \frac{\lambda_0^2}{c \Delta t} \]

Temporal coherence length

\[ l = c \Delta t \Rightarrow l = \frac{\lambda_0^2}{\Delta \lambda_0} \]

b) White light: 550 nm \( \rightarrow \) 750 nm \( \Rightarrow \) \( \Delta \lambda_0 = 400 \text{ nm} \)

\[ \lambda_0 = \frac{550 \text{ nm} + 750 \text{ nm}}{2} = 650 \text{ nm} \]

\[ \Rightarrow l = \frac{\lambda_0^2}{\Delta \lambda_0} \approx 0.76 \text{ \( \mu \)m} \]
2) Rectangular spectral distribution assumed.

\[ \ell = \frac{\lambda_0^2}{c \Delta \lambda} \]

\[ a) \quad \lambda_0 = 1550 \text{ mm} \quad \Delta \lambda = 15 \text{ mm} \quad \Rightarrow \ell \approx 16 \text{ mm} \]

\[ b) \quad \lambda_0 = 1550 \text{ mm} \quad \Delta \lambda = 3 \text{ mm} \quad \Rightarrow \ell \approx 800 \text{ mm} \]

\[ c) \quad \lambda_0 = 1550 \text{ mm} \quad \Delta \lambda = 0.1 \text{ mm} \quad \Rightarrow \ell \approx 24 \text{ mm} \]

\[ d) \quad \lambda_0 = 632.8 \text{ mm} \quad \Delta \nu = 1.5 \text{ GHz} \quad c \times 7.0 \cdot 10^7 \text{ m/s} \]

\[ \Delta \lambda = \frac{2 \lambda_0^2}{c \Delta \nu} \quad \Rightarrow \ell = \frac{2 \lambda_0^2}{c \Delta \nu} = c \Delta \nu = \frac{c}{\Delta \nu} \approx 0.2 \text{ m} \]

\[ e) \quad \Delta \nu = 100 \text{ MHz} \]

\[ \Rightarrow \ell = \frac{c}{\Delta \nu} \approx 3 \text{ m} \]
3) Coherence length: Michelson interferometer example

Michelson interferometer: red cadmium light illumination
- $\lambda_0 = 643.847\,\text{nm}$
- $\Delta\lambda = 0.001\,\text{nm}$

Two waves interfere within coherence length $l_c$

$$l_c = \frac{\lambda^2}{\Delta\lambda} \quad \text{(rectangular spectral profile assumed)}$$

Fringe pattern:

At $l_c$ the intensity is dropped to half of the maximum intensity.

Let's calculate the distance $d$ where the intensity is dropped to half.

$$2d = \frac{l_c}{2} \Rightarrow d = \frac{l_c}{4} = \frac{1}{4} \frac{\lambda^2}{\Delta\lambda} \approx 79.7\,\text{mm}$$

How many wavelengths: $\frac{d}{\lambda} \approx 124.1\times10^{-3}$
Coherence function for monochromatic light is of a form

\[ I = I_s + I_r + 2 \sqrt{I_s I_r} \cos(\Delta \phi) \]

- DC term
- Interference term

For broad band light source the coherence function is calculated by integrating over all wavenumbers

\[ I(\omega) = \int \left( I_s(d\omega) + I_r(d\omega) + 2 \sqrt{I_s(d\omega) I_r(d\omega)} \cos(2\pi \omega t) \right) \, d\omega. \]

Integration of first two terms leads to two stationary offsets. This offset is called DC component. Integration of the interference term can be done for example in MATLAB and results into an traditional interferogram.

Another way to show the form of coherence function:

Since the interference term is the autocorrelation of the emitted electric field, according to Wiener-Khinchin theorem the system spectrum and coherence function are a Fourier transform pair.

If the system spectrum has a Gaussian shape

\[ s(\omega) = e^{-\left(\frac{(\omega - \omega_0)^2}{2\Delta \omega^2}\right)} \]

where \( \omega_0 \) is the central wavenumber and \( \Delta \omega \) the 1/e half-bandwidth, the coherence
function is calculated by Fourier transform

\[ G(\Delta z) = \int_{-\infty}^{\infty} \tilde{S}(k) e^{-i2\pi k \Delta z} dk. \]

The result is another Gaussian function of a form

\[ G(\Delta z) = e^{-\Delta z^2 / (2 \Delta z^2)} \]

with FWHM corresponding to the coherence length.