Photonics 17 Risto Mortonen

1) a) $c = \nu \lambda_0 \Rightarrow \lambda_0 = \frac{c}{\nu} \Rightarrow \nu = \frac{c}{\lambda_0}$

optical frequency

Differentiation:

$$\Delta \lambda_0 = -\frac{c}{\nu^2} \Delta \nu = -\frac{c \lambda_0^2}{c^2} \Delta \nu = -\frac{\lambda_0^2}{c} \Delta \nu$$

let's drop sign out since Δ represents width and has no sign

Temporal coherence time

$$\Delta t = \frac{1}{\Delta \nu} \Rightarrow \Delta \lambda_0 = \frac{\lambda_0^2}{c \Delta t}$$

Temporal coherence length

$$l_c = c\Delta t \Rightarrow \underline{\underline{l_c = \frac{\lambda_0^2}{\Delta \lambda_0}}}$$

b) White light $350\text{ nm} \rightarrow 750\text{ nm} \Rightarrow \Delta \lambda_0 = 400\text{ nm}$

$$\lambda_0 = \frac{350\text{ nm} + 750\text{ nm}}{2} = 550\text{ nm}$$

$$\Rightarrow l_c = \frac{\lambda_0^2}{\Delta \lambda_0} \approx 0{,}76\ \mu m$$

2) Rectangular spectral distribution assumed.

$$l_c = \frac{\lambda_0^2}{\Delta\lambda_0}$$

a) $\lambda_0 = 1550\ nm$ $\quad \Rightarrow l_c \approx \underline{\underline{16\ \mu m}}$
$\Delta\lambda_0 = 150\ nm$

b) $\lambda_0 = 1550\ nm$ $\quad \Rightarrow l_c \approx \underline{\underline{800\ \mu m}}$
$\Delta\lambda_0 = 3\ nm$

c) $\lambda_0 = 1550\ nm$ $\quad \Rightarrow l_c \approx \underline{\underline{24\ mm}}$
$\Delta\lambda_0 = 0{,}1\ nm$

d) $\lambda_0 = 632{,}8\ nm$ $\quad c \approx 3{,}0 \cdot 10^8\ m/s$
$\Delta\nu = 1{,}5\ GHz$

$$\Delta\lambda_0 = \frac{\lambda_0^2}{c\Delta t} \Rightarrow l_c = \frac{\cancel{\lambda_0^2}}{\frac{\cancel{\lambda_0^2}}{c\Delta t}} = c\Delta t = \frac{c}{\Delta\nu} \approx \underline{\underline{0{,}2\ m}}$$

e) $\Delta\nu = 100\ MHz$

$$\Rightarrow l_c = \frac{c}{\Delta\nu} \approx \underline{3\ m}$$

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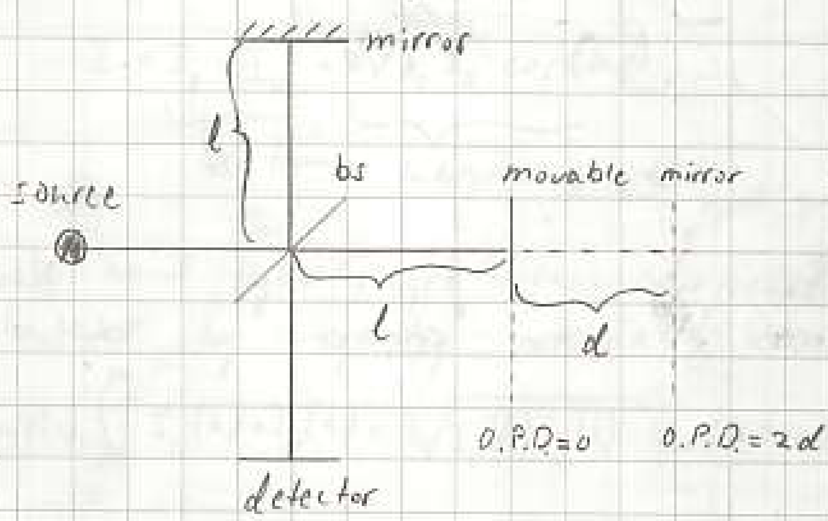
3) Coherence length: Michelson interferometer example

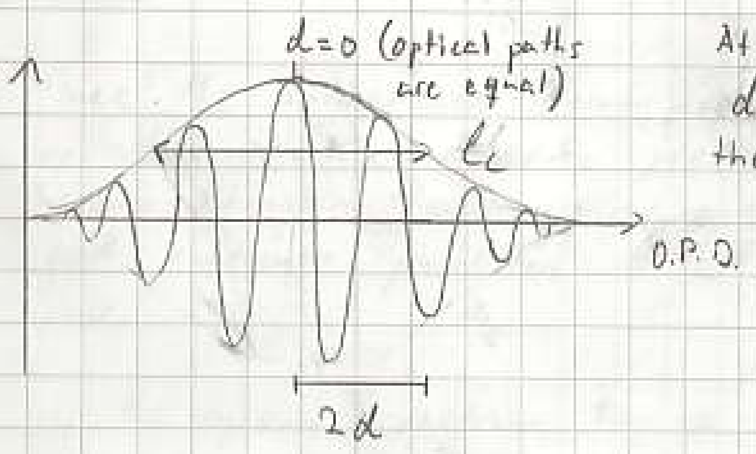
Michelson interferometer: red cadmium light illumination
- $\lambda_0 = 643{,}847\,nm$
- $\Delta\lambda_0 = 0{,}0013\,nm$



Two waves interfere within coherence length l_c

$$l_c = \frac{\lambda_0^2}{\Delta\lambda_0} \quad \text{(rectangular spectral profile assumed)}$$

Fringe pattern:



At l_c the intensity is droped to half of the maximum intensity.

Let's calculate the distance d where the intensity is droped to half.

$$2d = \frac{l_c}{2} \Rightarrow d = \frac{l_c}{4} = \frac{1}{4}\,\frac{\lambda^2}{\Delta\lambda} \approx \underline{\underline{79{,}7\,mm}}$$

How many wavelengths: $\frac{d}{\lambda} \approx \underline{\underline{124 \cdot 10^3}}$

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Bonus)

Coherence function for monochromatic light is of a form

$$I = \underbrace{I_s + I_R}_{\text{DC term}} + \underbrace{2\sqrt{I_s I_R}\,\cos(\Delta\varphi)}_{\text{Interference term}}$$

For broad band light source the coherence function is calculated by integrating over all wavenumbers

$$I(\Delta z) = \int_0^{\infty}\left[I_s(k) + I_R(k) + 2\sqrt{I_s(k) I_R(k)}\,\cos(2k\Delta z)\right] dk$$

Integration of first two terms leads to two stationary offsets. This offset is called DC component. Integration of the interference term can be done for example in matlab and results into an traditional interferogram.

Another way to show the form of coherence function:

Since the interference term is the autocorrelation of the emitted electric field, according to Wiener-Khinchin theorem the system spectrum and coherence function are a Fourier transform pair.

If the system spectrum has a Gaussian shape

$$S(k) = e^{-\frac{(k-k_0)^2}{\Delta k^2}},$$

where k_0 is the central wavenumber and Δk the "1/e" half-bandwidth, the coherence function is calculated by Fourier transform

$$G(\Delta z) = \int_{-\infty}^{\infty} S(k) e^{4\pi i \Delta z k} \, dk.$$

The result is another Gaussian function of a form

$$G(\Delta z) \propto e^{-\Delta z^2 \Delta k^2}$$

with FWHM corresponding to the coherence length.