Optics, Spring 2016

Exercise 7, 15.3.2016

Problem 4 is worth 2 points

1.State of polarization (Ch. 8.1)

a) Describe completely the state of polarization (in Hecht's convention), if the electric field is:
i) $\bar{E} = \hat{x}E_o \sin\left[2\pi\left(\frac{z}{\lambda} - ft\right)\right] - \hat{y}E_o \sin\left[2\pi\left(\frac{z}{\lambda} - ft\right)\right]$
ii) $\bar{E} = \hat{x}E_o \cos(\omega t - kz) - \hat{y}E_o \cos(\omega t - kz + \pi/2)$
iii) $\bar{E} = \hat{x}E_o \sin(\omega t - kz) - \hat{y}E_o \sin(\omega t - kz + \pi/4)$.

2. Linear polarizers (Ch. 8.1)

a) A beam of linearily polarized light (intensity I_0) passes through two ideal linear polarizers aligned to have their axes of polarization perpendicular to each other. What is the transmitted intensity?
b) A third polarizer is placed between the two, with the axis of polarization rotated 30°in respect to axis of the first polarizer. What is the transmitted intensity?

3. Birefringence (Ch 8.4, 8.7)

a) A calcite crystal ($n_o = 1.6584,\ n_e = 1.4864,\ \lambda = 584.3\ nm$) is polished in a way that the optical axis is perpendicular to the surface. A ray of light is incident at the surface at $\theta = 50°$. Compute the frequency, wavelength and angle refraction for o and e waves.
b) What is the minimum thickness for a quartz ($n_o = 1.5443,\ n_e = 1.5534,\ \lambda = 584.3\ nm$) retarder if it is to be a quarter wave plate?

4. Wave plates and polarizers (Chs. 8.1, 8.7)

a) Plane wave $\bar{E} = \bar{E}_0 \cos(\omega t - kx)$, $\bar{E}_0 = [0, E_{oy}, E_{0z}]$ is incident at an ideal waveplate (phase change $\Delta\phi$). The angle between the direction of polarization and the optical axis of the waveplate is α. The optical axis is parallel to the z axis. An ideal polarizer is placed after the wave plate, with the angle between the axis of polarization and z axis being β. Show that the intensity after the polarizer is:

$$I = I_0\left[(\sin\alpha\sin\beta + \cos\alpha\cos\beta)^2 - 4\sin\alpha\sin\beta\cos\alpha\cos\beta\sin^2\left(\frac{\Delta\phi}{2}\right)\right]$$

Hint: calculate the time-averages in the usual way to get I.
b) Show that in the case of a half wave plate and $\alpha = \pi/4$, the output intensity is unchanged and the state of the polarization is preserved.