Optics, Spring 2016

Exercise 4, 16.2.2016

These problems are related to Hecht 4th ed. chapter 5.6. Use e.g. Matlab for numerical calculations.

1. Optical fibers

The refractive indices for the cladding and core of an optical fiber are n_c and n_f, respectively.
a) Derive the equation for maximum acceptance angle θ_{max}
b) Compute i) numerical aperture, ii) θ_{max} and iii) the number of propagating modes ($n_f = 1.51$, $n_c = 1.48$, $d_{core} = 70\ \mu m$, $\lambda_0 = 1.3\ \mu m$).

2. Group index

Group velocity is defined as: $v_g = \frac{d\omega}{dk} = \frac{c}{N_g}$.
Show that, the group index can be written as: $N_g = n - \lambda \left(\frac{dn}{d\lambda}\right)$. Hint: think about what depends on the refractive index and use the chain rule.

3. Modeling group index

The wavelength dependent refractive index can be modeled using the Sellmeier equation:

$$n^2 = 1 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2} + ...$$

For pure silica, the first three coefficients are (wavelengths in µm):

$$A_1 = 0.696749, \quad A_2 = 0.408218, \quad A_3 = 0.890815$$
$$\lambda_1 = 0.0690660, \quad \lambda_2 = 0.115662, \quad \lambda_3 = 9.900559$$

Calculate the group index (numerically) from 500 nm to 1.8 µm.

4. Material dispersion

Calculate the broadening of an optical pulse in 1km long pure silica fiber due to material dispersion for an LED operating at 850 nm (linewidth 20 nm). Hint: derive $\frac{dt}{d\lambda}$ in terms of the refractive index. Use the definition of group velocity when needed. You will need some numerical data from prob. 3.