Optics, Spring 2016

Exercise 2, 2.2.2016

Problem 2 is worth 2 points.

1. Harmonic waves

a) Explain, why a plane wave represented by$\tilde{\psi}\left(\bar{r},t\right)=\exp\left(-i\left(\omega t\mp\bar{k}\cdot\bar{r}\right)\right)$ propagates in the direction of $\pm\bar{k}$.
b) Show, that: $\frac{\partial^2\psi}{\partial r^2}+\frac{2}{r}\frac{\partial\psi}{\partial r}=\frac{1}{r}\frac{\partial^2(r\psi)}{\partial r^2}$
c) BONUS (1 extra point): Show that the general solution of the one dimensional wave equation is $\psi\left(x,t\right)=f\left(x-ct\right)+g\left(x+ct\right)$, where f and g are arbitrary twice differentiable functions.

2. The Fresnel equations

Electromagnetic plane wave is incident at an interface between two dielectric media (refractive indices n_1 and n_2, assume that $\mu_r=1$). The amplitude of the incident electric field E_{0i} is known. Derive the Fresnel equations describing the reflected and transmitted electric field amplitudes E_{0r} and E_{0t} when the incident electric field is parallel to the plane of incidence.
Hint: Follow the description in Hecht, 4th edition, chapter 4.6.2. Explain every intermediate step. Do not just blindly copy the book chapter.

3. Reflectance and transmittance

a) Derive the expressions for the amplitude coefficients given by problem 2 for normal incidence.
b) Derive the expression for coefficients of transmittance and reflectance at normal incidence:

$$\begin{aligned} R &= \left(\frac{n_2-n_1}{n_2+n_1}\right)^2 \\ T &= \frac{4n_1n_2}{\left(n_2+n_1\right)^2} \end{aligned}$$

c) What is the percendate of reflected irradiance at an air-glass interface (normal incidence, $n_{air}=1$,$n_{glass}=1.5$)?

4. Brewster's angle

Light in incident at a interface between two dielectric media. The incident angle in called Brewster's angle (a.k.a. the polarization angle) when the sum of incident and refracted angles is 90° ($\theta_i+\theta_t=90°$).
a) Calculate the angle at an air-glass interface using Snell's law.
b) Calculate the coefficient of reflected amplitude incident at Brewster's angle when the electric field is polarized in the direction parallel to the plane of incidence.