

Optics, Spring 2016

Exercise 1, 26.1.2016

1. Complex number representation

- Show that multiplying a complex number by $\pm i$ is equivalent to phase shift of $\pm 90^\circ$.
- Two waves of the same amplitude, frequency and speed are overlapping in some region of space:

$$\psi(y, t) = A \cos(ky + \omega t) + A \cos(ky - \omega t + \pi)$$

Show using complex exponentials that:

$$\psi(y, t) = -2A \sin ky \sin \omega t$$

2. Maxwell's equations and waves

- Derive the the free space wave equation for electric and magnetic fields from Maxwell's equations.
- How does the situation change in linear dielectric medium?
- What is the physical basis of the assumption that in typical dielectrics $\mu_r = 1$.

3. Three dimensional waves

Harmonic plane wave propagates in linear dielectric with homogenous ϵ and $\mu_r = 1$. Derive the expression for magnetic field (amplitude, phase and polarization), if the complex electric field is linearly polarized in x-direction:

$$\mathbf{E} = E_0 e^{-i(\omega t - kz)} \hat{x}$$

4. Refraction and reflection

- Derive Snell's law using Fermat's principle.
- A ray incident at θ_i to a planar glass plate exits the plate at θ_o (see the figure). Show that $\theta_i = \theta_o$. The plate is immersed in air.
- Derive the expression for the displacement a if the thickness of the plate is d .

